

“To find one’s way around the mathematical terrain, it is important to see how the various representations connect with each other, how they are similar, and how they are different. The degree of students’ conceptual understanding is related to the richness and extent of the connections they have made” (National Research Council, 2001).

This unit begins by investigating two major place value ideas:

- Grouping objects by tens to count them (e.g., 4 groups of ten and 2 ones left over); and
- Using language to make connections among groupings of tens and ones (4 tens and 2 ones), their names (forty-two), and standard symbols (42).

The first four lessons focus on this content to build understanding of our place value system. In Lesson 4 *Spin for 50*, students make the transition from calling the ungrouped counters “leftovers” to calling them “ones.”

Lesson 5 helps students recognize the connection between tens and ones and the sequence of whole numbers to 100. The 100 Chart illustrates the connection between, for example, the number 34 and groups of 3 tens and 4 ones. Students discuss emerging place value patterns.

In Lessons 6 and 7, students use their work with tens and ones to begin an examination of number relationships.

Lesson 8, *Full of Beans*, is a lab in which students explore volume by filling containers with nonstandard units. The investigation enables students to apply their knowledge of grouping by tens while building concepts of volume. In particular, students investigate the inverse relationship between the size of the unit and the measurement of the volume.

Throughout the unit, students build number sense by exploring relationships among numbers and the relative size of numbers. To develop these concepts they represent groups of tens and ones using ten frames, 100 Charts, number lines, links, manipulatives, and drawings. Making connections among these representations and the two-digit numbers they represent will help solidify these place value ideas.

Development of Base-Ten Concepts. Students develop the concept of number by first relying on unitary counts (counting by ones) to understand quantities. For example, they count 27 objects and write and say 27. To develop place-value concepts, students first need to recognize that a group of ten can be counted as a unit. Students also need to recognize that a group of ten is made up of ten individual units. Meaningful use of the groups of ten to represent quantities does not develop easily or quickly. Rather, this requires the integration of language and counting strategies.

Counting plays a key role in helping students construct base-ten concepts about quantity. Students use a variety of counting strategies that help them think about the quantity in different ways:

1. *Counting by ones.* Students count objects that are grouped and not grouped by one to determine the quantity.
2. *Counting by groups and the leftovers.* Students count the groups and the leftovers and integrate the counts to determine the quantity.
3. *Counting by tens and ones.* Students skip count and count on to determine the quantity.

Though these connections seem obvious to adults, these concepts and procedures are difficult for students to construct and students need to construct them for themselves. Students also need to convince themselves that no matter how a quantity is counted (grouped or not grouped) the quantity will not change.

Connect Representations. The symbols used for writing numbers and the conventions for saying numbers must also be integrated into these counting experiences. When a student counts by tens and ones they say the number of groups and singles separately, e.g., five tens and seven. This helps students easily transition to saying fifty-seven. Students should also reiterate that fifty-seven is 5 groups of ten and 7 singles or ones. Students also need to make explicit connections between the groupings and the symbols, 57.

Math Facts and Mental Math

Addition Facts. Students work on developing fluency with the addition facts with sums to ten and developing strategies for the related fact families in Group C.

Group C: $1 + 9$, $2 + 7$, $2 + 8$, $3 + 6$, $3 + 7$, $4 + 6$, $5 + 5$

See Mathematics In This Unit in Unit 6 for more about addition facts development in Grade 1.

These facts involve identifying the partitions of ten and using ten to solve facts near ten. For example, $8 + 2 = 10$ so $7 + 2 = 9$ or one less.

Resources

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