Unit 1

MATHEMATICS IN THIS UNIT
Sampling and Classifying

Taken from the *Math Trailblazers* digital Teacher Guide

In this unit, students gather data to make predictions and generalizations about a population. Similar to investigations in earlier grades, students will represent variables in a drawing, collect data, and represent that data to describe and make predictions about a population. Students analyze the length of students' first names and the kinds of beans in a "bean" population.

This population context gives students an authentic reason to use mathematical models such as graphs and tables to represent and reason about a situation.

Populations and Sampling. The context for this unit is the study of populations. A **population** is a group of persons or things from which samples are taken for statistical measurement. In everyday language, we talk about the population of Illinois or Topeka. However, we can also talk about the population of leaves on the General Sherman giant sequoia tree or the population of deer in a forest preserve.

Some populations are so small and so well defined that we can count each individual member. For instance, the population of giant pandas in U.S. zoos can be counted or measured directly. More often, it is impossible, impractical, or unnecessary to count or measure an entire population. The population of giant pandas in the wilds of western China, for example, cannot be counted exactly. When an entire population cannot be studied directly, we can often gather useful information about it through a method called sampling.

A **sample** is a subset of a population. If you pull five leaves from a tree, you have a sample from the population of leaves on that tree. If you pull a handful of jelly beans from a bowl, you have sampled the jelly bean population of that bowl. Studying a sample often allows us to draw probable conclusions about the population under study. If three of five oak leaves show evidence of blight, it is likely that the tree has a problem. If half of a handful of jelly beans is red, it is probable that a substantial portion of the beans in the bowl are also red.

Sampling is worth studying both for its applications (public opinion polling, statistical process control, and wildlife population estimation) and for the interesting mathematics it requires (variables, classification, probability, mathematical reasoning, and statistics). Students begin studying populations and sampling in first grade when they survey students about their favorite colors. In each subsequent grade, population studies in the form of surveys and classification labs are carried out. The activities in this unit are part of this series. The breadth and power of the mathematics of such population studies make a rich beginning for what we hope will be a most productive school year.

TIMS Laboratory Method. This unit reviews the four steps of the TIMS Laboratory Method: drawing a picture, collecting the data, graphing the data, and analyzing the results.

The four steps of the TIMS Laboratory Method will be familiar to students who have used *Math Trailblazers* before. Others will become accustomed to the routines as they gain experience with the method.

Students eventually will apply this method on their own but may need to be guided at first. One of your more difficult instructional decisions will be how much guidance to give and how to balance imitation and autonomy. Learning by imitation will make for more orderly lessons, not an unimportant consideration in the beginning of the year. On the other hand, too much imitation can undermine student autonomy and can foster misconceptions about what mathematics and science are. After all, what really matters in the long run is what students can do without teacher guidance.

Lesson 1 *First Names* accommodates a range of approaches from teacher-directed to open-ended. In the main part of the lesson, you can be as directive as you think appropriate, especially since there is one data set for the whole class rather than a separate data set for each group. On the other hand, the homework and extensions present opportunities for students to design and carry out their own investigations. You can adapt this lesson to your class needs while you are getting to know your students.

Lesson 3 *Kind of Bean,* which incorporates all four steps of the TIMS Laboratory Method, demands more student independence since students gather their own data. Although imitation is still important, data will vary from student to student—therefore, methods, but not results, will be imitated. You can have some interesting discussions when the same method yields different results.

One aspect of the method you may want to stress in these early experiments is how the real objects, the picture, the data table(s), and the graph(s) all represent the same situation. Students will benefit from discussions connecting and comparing these various representations.

Variables and Values. Variables are an important part of both mathematics and science. Several basic procedures for handling variables are involved in this unit: distinguishing between variables and values, denoting variables by symbols, and labeling data table columns and graph axes with variable names.

Do not delay stressing the idea that an experiment is an investigation about relationships between variables. *First Names* can be seen as a study of how many students (*S*) have each number of letters (*L*) in their first names. *Kind of Bean* is a study of the relationship between the number of beans (*N*) and of each kind of bean (*K*).

Number Lines. The number line representation is used in the earlier grades, and students will use them extensively throughout the year as they develop whole number concepts and procedures. In Lesson 2, students use the Number Line Target game to review the use of this representation.

Algebra in the Early Grades

There is a debate among mathematicians and educators about the content of algebra and especially the content of algebra in the early grades. Some have defined algebra as modeling, as pattern finding, as the study of structure, and as making sense of one's world quantitatively.

> "We advocate an early emphasis on developing children's ability to conceive of, reason about, and manipulate complex ideas and relationships, as an equal complement to numerical reasoning and computation. Children who develop a rich capacity for reasoning about general relationships among quantities will possess the conceptual foundation for learning and making sense of different programs and views of algebra."
>
> (John P. Smith III and P. Thompson, 2008)

This unit creates a context for students to reason quantitatively about populations by looking for, describing, and making predictions from patterns in categorical data represented in tables and graphs. This focus on patterns captures the most central ideas of early algebraic reasoning. As such, the activities in this unit form a bridge between the data strand and the algebra strand in *Math Trailblazers*.

This focus on early algebraic thinking is developed throughout *Math Trailblazers.* Following recommendations of researchers, activities are integrated through all grades that give students concepts and skills needed to make a natural transition from learning and doing arithmetic to learning and doing algebra (Carpenter et al., 2003; Carraher and Schliemann, 2007; Kaput, 2008; Kilpatrick and Izsak, 2008; Schliemann et al., 2007). Students extend and connect their concepts of number, geometry, measurement, and data to develop tools for algebraic reasoning. These tools enable them to "do algebra," that is, to identify, describe, visualize, and simplify patterns and relationships. They learn to generalize procedures while they use arithmetic. They also learn to make generalizations about numbers that are collected in data sets, organized in tables, and pictured in graphs.

> "Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved, attending to the meaning of quantities . . ."
>
> (Common Core State Standards Initiative, 2010)

Review and Practice

Every unit includes opportunities for distributed practice of concepts and skills. These resources can be found primarily in two places:

- Daily Practice and Problems (DPP) in the *Teacher Guide*
- Home Practice in the *Teacher Guide*

Daily Practice and Problems is a set of short exercises that provide ongoing review and study of math concepts and skills and provides a structure for systematically reviewing basic math facts. See the Daily Practice and Problems for this unit for more information about how they are organized and for ways to incorporate DPP items into your daily routine.

The Home Practice section of the *Teacher Guide* is a series of problems that supplement the homework included in the lessons. The Home Practice distributes skills practice throughout the units and reviews concepts studied in previous units.

Addition Facts. In this unit, students review their addition facts. Students have been developing strategies for solving the addition facts since their early years and are fairly close to gaining or have gained proficiency. In Unit 2, students will use their addition facts to develop strategies and proficiency with the related subtraction facts. The addition facts are sorted into groups by strategy. See Figure 1.

Group	Addition Facts	Strategy Used
A	0 + 1, 1 + 1, 2 + 1, 3 + 1, 0 + 2, 2 + 2, 3 + 2, 4 + 2	Counting and Zero
B	3 + 0, 4 + 0, 4 + 1, 5 + 1, 6 + 1, 5 + 2, 6 + 2, 5 + 3, 7 + 1, 1 + 8	Counting and Zero
C	1 + 9, 2 + 7, 2 + 8, 2 + 9, 3 + 6, 3 + 7, 3 + 8, 4 + 6, 4 + 7, 5 + 5, 5 + 6	Making Tens
D	3 + 3, 3 + 4, 4 + 4, 4 + 5, 6 + 6, 6 + 7, 7 + 7, 7 + 8, 8 + 8, 10 + 9, 10 + 10	Using Doubles
E	5 + 7, 8 + 4, 8 + 5, 9 + 3, 9 + 4, 9 + 5, 10 + 1, 10 + 2, 10 + 3	Using Tens
F	8 + 6, 9 + 6, 9 + 7, 10 + 4, 10 + 5, 10 + 6, 10 + 7, 10 + 8, 9 + 8, 9 + 9	Using Tens

Figure 1: *Addition Facts Groups reviewed in Grade 2*

Resources

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