Unit 7

MATHEMATICS IN THIS UNIT
Subtracting Larger Numbers

Taken from the *Math Trailblazers* digital Teacher Guide

Subtraction Methods. In this unit, students develop meaningful, accurate, and efficient methods for multi-digit subtraction. The unit includes opportunities for students to practice both addition and subtraction in various contexts so that they learn to choose computation methods based on the context and the numbers in the problem.

The development of computation methods in *Math Trailblazers* is based on research that corroborates the relationship between conceptual understanding and procedural fluency (Fuson & Burghhardt, 2003; Fuson, 2003; Verschaffel, et al., 2007; National Research Council, 2001; NCTM, 2000; Hiebert & Wearne, 1996). In this unit, students develop their own methods for subtraction to first solve word problems and then to solve problems with larger numbers using base-ten pieces. With these opportunities, students make connections between their understanding of place value and procedures for subtraction. As stated in *Adding It Up: Helping Children Learn Mathematics,* "Opportunities to construct their own procedures provide students with opportunities to make connections between the strands of proficiency. [The strands of mathematical proficiency include conceptual understanding, procedural fluency, and productive disposition (National Research Council, 2001, p. 5).] Procedural fluency is built directly on their understanding. The invention itself is a kind of problem solving, and they must use reasoning to justify their invented procedure. Students who have invented their own correct procedures also approach mathematics with confidence rather than fear and hesitation" (Kamii and Dominick, 1998, quoted in National Research Council, 2001, p. 197).

Throughout the unit, students share their own strategies with their classmates and read about possible strategies in the *Student Guide.* See Figure 1. Comparing the various strategies and trying them out allows students to analyze the efficiency and usefulness of the methods in different situations (NCTM, 2000, p. 153).

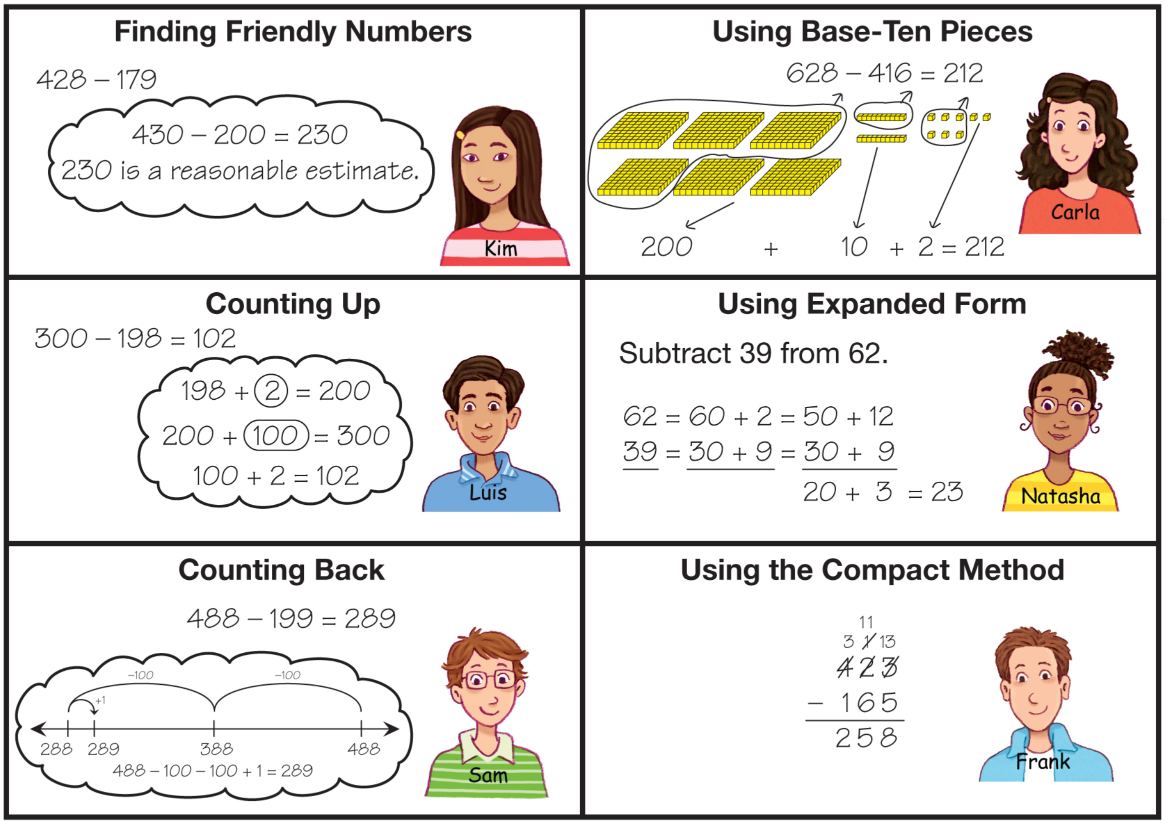


Figure 1: *A* Subtraction Strategies Menu *discussed in this unit*

Developing Mental Math

Partitioning Numbers. The ability to decompose and recompose numbers helps students understand more deeply the meaning of the computational algorithms they use and provides a greater flexibility in using them. The partitioning of numbers by place value (e.g., 1234 = 1000 + 200 + 30 + 4) is especially important when computing with larger numbers, both when using an algorithm and when using mental math.

For example, when using the traditional algorithm to solve the subtraction problem 1234 – 173, a trade is required. When the trade is made correctly, the minuend (top number) does not change in value. Rather, a different partition of the number is used to make subtraction possible.

$$1234 = 1000 + 200 + 30 + 4$$
$$1234 = 1000 + 100 + 130 + 4$$

	1234 = 1000 + 100 + 130 + 4
Subtract 173	100 + 70 + 3
	1000 + 0 + 60 + 1 = 1061

Figure 2: *Subtract using Expanded Form*

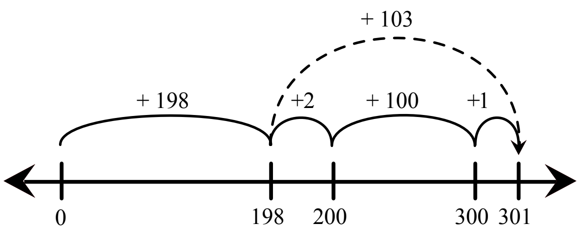
Without this understanding, a student who does not realize that the trade does not (and cannot) change the value of the top number (minuend) may make random changes to the top number in order to make it easy to subtract.

Mental Math Strategies. Development of mental math strategies is an important outcome of students' study of whole numbers for two reasons. First, use of these strategies helps students in classrooms develop number sense and flexibility with numbers. Secondly, using mental math in the real world is often the most efficient means of solving problems, either for finding exact answers or for finding estimates when appropriate. Therefore, students need to have many opportunities to choose an efficient strategy for a given problem based on the numbers in the problem and the real-world context. For example, using mental math strategies to find exact answers to the subtraction problems in Figures 3 and 4 is more efficient than subtracting across zeros using the traditional algorithm.

3001 – 2899 = 102 using a counting-up strategy:

$$2899 + \text{(1)} = 2900$$
$$2900 + \text{(100)} = 3000$$
$$3000 + \text{(1)} = 3001$$
$$1 + 100 + 1 = 102$$

Figure 3: *Choosing an efficient method to subtract*



301 = 198 + 2 + 100 + 1
301 = 198 + 103

Therefore,
301 – 198 = 103

Figure 4: *Using flexible partitioning to solve 301 – 198*

Students should have opportunities to develop strategies that make sense to them. Some strategies may include some mental calculations along with a few written notes. For example, to solve 301 – 198 in Figure 4, students may visualize the number line but keep track of the hops by writing down the numbers: 2 + 100 + 1 = 103.

Discussing their strategies and explaining them in writing will help students clarify their own thinking and provide new ideas for others. However, it is important that students have opportunities to practice strategies that emphasize their efficiency. That is, when they choose to solve a problem mentally, they should not have to explain their thinking every time. For these reasons, students will see different types of directions for solving mental math problems in the unit. In some cases they are asked to explain their thinking in writing. At other times they are asked to share their thinking with a partner in discussion or to choose one problem out of a set of problems to show their reasoning in writing.

Choosing Appropriate Strategies. Research in *Math Trailblazers* classrooms using previous editions indicates that students need support developing and using multiple computation strategies, instead of relying heavily on one procedure. Therefore, we have developed Strategy Menus for each operation to help students select appropriate methods for solving problems based on the context of the problem or the numbers involved. See Figure 1 for a menu of subtraction strategies from Lesson 5.

The menus remind students of strategies and methods they have practiced in class and help them choose appropriate methods for calculating based on the numbers in the problems. For example, students will arrive at accurate solutions more often if they have the flexibility to solve the subtraction problem 3001 – 2899 using a counting-up strategy, than if they use the traditional algorithm to subtract across zeros. See Figure 3. On the other hand, it is more appropriate to use the algorithm to solve 9524 – 3679.

Developing Paper-and-Pencil Methods for Subtraction. In this unit, students learn the algorithm that is traditionally taught in the United States. To use the algorithm efficiently and accurately, students need a solid understanding of place value in the base-ten system. Therefore, students build on their previous experiences with base-ten pieces to develop meaning for the algorithm. Base-ten pieces are effective if students are given enough time to think for themselves how to compose and decompose numbers in order to take away a quantity. Then, students need to connect the process with the base-ten pieces to written symbols (National Research Council, 2001, p. 198; Fuson and Burghardt, 2003, 289–290).

From our research we have found that students sometimes make the errors shown in Figure 5. Similar errors were found in other research studies (Fuson and Burghardt, 2003, p. 286)

Error A	Error B	Error C
	2 12 11	2 10 11
7220	~~3~~ ~~2~~ ~~1~~	~~3~~ ~~0~~ ~~1~~
– 6965	– 2 6 8	– 1 8 9
1745	6 3	1 2 2

Figure 5: *Possible subtraction errors*

In A, the student subtracted the smaller number from the larger in each column, possibly thinking only of the individual digits instead of the value of each place. In B and C, the students did not trade correctly. Students who make these mistakes often do not understand that when they make trades the value of the top number, the minuend, should not change. Students should be able to look back at their trades and decide if the new partition represents the same number. For example, writing a number sentence for the trades recorded for B (200 + 120 + 11 = 331), shows that the new groupings equal 331 instead of 321. Asking students to think about the value of the digits and to link their written method with base-ten pieces will often help students correct their own errors (Fuson and Burghardt, 2003, p. 289).

Students often make the mistake in Error C when they must trade across zeros. When students have multiple strategies to choose from, they can solve similar problems using other efficient strategies that may result in greater accuracy. For example, students can first subtract 189 from 299 and then add 2 (299 – 189 = 110; 110 + 2 = 112), or they can count up from 189 (189 + (1) = 190, 190 + (10) = 200, 200 + (101) = 301, so the answer is 1 + 10 + 101 = 112.) See the Counting Up strategy in Figure 4.

Math Facts and Mental Math

Subtraction Facts. Students review and are assessed on the subtraction facts in Groups 1–4 to maintain and increase fluency and to learn to apply subtraction strategies to larger numbers. See Mathematics In This Unit in Unit 2 for more about subtraction facts development.

Multiplication Facts. Students work on developing number sense for the multiplication facts for the last six facts (4×6, 4×7, 4×8, 6×7, 6×8, 7×8) in this unit by writing a story, drawing a picture, and writing number sentences for math facts. See Figure 6. For more about the multiplication facts development see Mathematics In This Unit in Unit 3.

$$4 \times \square = 32$$

An octopus has 8 legs.
This group of 4 has 32 legs.

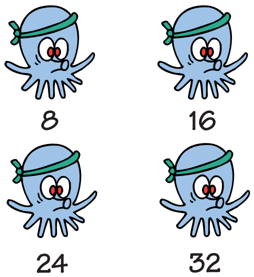


Figure 6: *Example story and picture for $4 \times \square = 32$*

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