Unit 11

MATHEMATICS IN THIS UNIT
Analyzing Shapes

Taken from the *Math Trailblazers* digital Teacher Guide

"Given children's affinity toward, knowledge of, and ability to gain geometric knowledge, it is important that this domain of mathematics not be neglected. Instruction in geometry needs to complement the study of number and operation in pre-K to 8" (National Research Council, 2001). *Math Trailblazers* echoes this expectation by emphasizing the importance of geometry in the mathematics curriculum. This unit represents the focal point of geometry in third grade. Students describe, analyze, and classify two-dimensional and three-dimensional shapes using their properties. Students also discover relationships within and among these properties as they advance their understanding through stages, from basic intuition to analysis and informal deduction.

van Hiele Levels of Geometric Development

Much of the approach to geometry found in the *Math Trailblazers* curriculum is grounded in the insights of Dutch educators Pierre van Hiele and Dina van Hiele-Geldof. Ongoing research in mathematics education continues to confirm the five levels of geometric development first described by the van Hieles in the 1950s (Burger and Shaughnessy, 1986). The five levels are:

Level 0: Visualization. Students judge geometric objects by their appearance, but not by attributes. For example, a student can identify a rectangle because it "looks like a rectangle," but not because it has opposite sides equal and four right angles.

Level 1: Analysis. Students begin to describe the properties of objects. A figure is no longer judged because it "looks like one," but rather because it has certain properties. For example, an equilateral triangle has three equal sides, three equal angles, and line symmetry.

Level 2: Informal Deduction. Students logically order the properties of figures and are able to deduce that one property precedes or follows from another property. They see relationships among figures. For example, a square has all the properties of a rectangle; therefore, a square is a rectangle. Students may also be able to define a square based on its properties.

Level 3: Deduction. Students write formal proofs based in an axiomatic system. A rigorous high school geometry course is taught at Level 3.

Level 4: Rigor. Students can work with different axiomatic systems. This level corresponds to college work in geometry (Crowley, 1987; van Hiele, 1999).

The van Hieles found that each level, while not age specific, builds on the previous level. Students proceed from level to level sequentially and no level can be omitted. Advancement depends on content and method of instruction (Crowley, 1987; van Hiele, 1999). Moreover, a student's experiences with lower-level reasoning at the elementary school level are critical to success with geometry in later schooling. Students who are at Level 0 or 1 when entering high school geometry have a poor chance of success. Students who begin high school geometry at Level 2 have at least a 50% chance at succeeding (Senk, 1989). Unfortunately, many upper elementary students are still at Level 0. This is not surprising, as researchers have found that most geometry questions asked in standard elementary math textbooks were answerable with Level 0 understanding (Fuys, Geddes, and Tischler, 1988).

In this unit, we start with Level 0 ideas by asking students to identify and draw various geometric figures. Most of the work in this unit is at Level 1, where students describe the properties of two-demensional and three-dimensional shapes. Level 2 ideas are introduced as students explore ways to classify these figures.

The content in this unit focuses on the two-dimensional pieces of a tangram puzzle and then three-dimensional solids. Students analyze and classify quadrilaterals and then polyhedrons, cylinders, and cones by describing and making connections among their properties.

Words, Words, Words

Vocabulary boosting is not a major goal of *Math Trailblazers®,* but effective communication is, and words are important for that. Too often, traditional mathematics texts turn geometry into a dreary parade of terms and definitions. Appropriate terminology can enhance both communication and understanding; children also enjoy learning and using fancy words. The issue is balance; use your professional judgment.

The Vocabulary in this Lesson section in the Lesson Guides lists minimal requirements. We cannot, for example, imagine talking about the parts of a triangle unless *side* and *vertex (corner)* are understood. Other technical terms are used in the lessons but are not as critical. We leave to your discretion the degree of familiarity you want your students to have with such terms.

When you talk about vocabulary with your students, distinguish mathematical English from everyday English. In mathematics, we often seize upon everyday terms and give them precise meanings that may differ from common meanings. Then, ironically, we complain loudly when the terms are used in their everyday sense. In everyday language, for example, a square is not a rectangle—it's a square. For most people, "rectangle" means "nonsquare rectangle." When we insist that "A square is a rectangle," students who are thinking only of vernacular meanings may conclude that mathematics is nonsense. By distinguishing mathematical usage from everyday usage, you may avoid this outcome.

Remember also that communication is more than vocabulary. Several times in this unit, for example, students are asked to explain how they know that all shapes that can be made with a given number of triangles have been identified. At other times, students are asked to describe how they solved a problem. Effective mathematical communication includes clearly describing procedures and results and arguing convincingly.

Math Facts and Mental Math

Multiplication Facts. This unit continues a systematic review and assessment of students' fluency with the multiplication facts. In this unit, students will have opportunities to gain fluency with the multiplication facts for the nines. Students should be encouraged to reason from facts they know and to break the factors into factors they know. For example, to solve 9×4 is $9 \times 2 + 9 \times 2 = 18 + 18$, or 36.

Resources

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