Measurement and Patterns

Taken from the *Math Trailblazers* digital Teacher Guide

Meeting the Algebra Standard in the Elementary Grades

This unit builds on students' previous knowledge of arithmetic, coordinate geometry, data, and measurement. By making connections among these strands of mathematics, students deepen their knowledge of each of these strands that are addressed in the Number and Operations, Geometry, Data and Probability, and Measurement Standards as outlined in the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (NCTM, 2000). Furthermore, they develop concepts that form a foundation for the study of algebra. The Algebra Standard for grades 3–5 calls for students to be able to:

Understand patterns, relations, and functions

- describe, extend, and make generalizations about geometric and numeric patterns;
- represent and analyze patterns and functions, using words, tables, and graphs.

Represent and analyze mathematical situations and structure using algebraic symbols

- identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers;
- represent the idea of a variable as an unknown quantity using a letter or a symbol;
- express mathematical relationships using equations.

Use mathematical models to represent and understand quantitive relationships

- model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.

Analyze change in various contexts

- investigate how a change in one variable relates to a change in a second variable;
- identify and describe situations with constant or varying rates of change and compare them.

Many of these ideas are addressed in this unit using skills and contexts appropriate for third graders.

Graphing. In Lessons 1–3, students learn to use coordinates and ordered pairs to locate objects on maps and points on graphs. They learn to label the horizontal axis as the left/right or x-axis and the vertical axis as the front/back or y-axis. They apply their knowledge of length measurement and multiplication to the use of scales on maps. Lesson 3 uses the context of mapping the sea floor to look for lost treasure in the "Ghost Galleons" story to develop the idea of looking for patterns in point graphs and drawing best-fit lines to delineate the pattern.

Expressing Relationships Using Symbols. In Lesson 4, students review how to measure mass with a two-pan balance. Using this context, students use the less than (<), greater than (>), and equal (=) signs to write number sentences. For example, students place one 5-gram and two 1-gram masses in one pan and one 10-gram mass in the other and write the sentence: $7 < 10$. They then use the balance to complete number sentences such as $7 \times 5 = 3 \times 10 + \square$. Students model the problem by placing seven 5-gram masses in one pan and three 10-gram masses in the other. They then find what must be added to one pan to balance the other.

Patterns. In Lessons 3–6, students describe, extend, and make generalizations about numeric patterns in data. Lesson 5 is the laboratory investigation *Mass vs. Number*. In this lab they measure the mass of different numbers of identical objects and record the data in a table and graph. They represent the variables in the lab with symbols (*M* for mass and *N* for Number of Objects) and investigate how a change in one variable relates to a change in the other. See Figure 1. For example, if the number of objects doubles, what happens to the total mass? If the number of objects increases by one, what happens to the total mass of objects? In Lessons 5 and 6, students use patterns in data as represented in both tables and graphs to solve problems.

N Number of Trapezoids	*M* Mass (in g/unit)
1	6
2	11
3	16
4	21
6	33

Figure 1: *A data table for finding the mass of pattern block trapezoids*

Best-Fit Lines. A major goal of both mathematics and science is to find patterns in data and to use them to make predictions. The graph and the data table are two powerful tools for helping us find patterns and make predictions. In Unit 10 *Exploring Multiplication,* we saw that some investigations yield graphs with data points that lie exactly on a straight line. When there is measurement error, the data points often are close to, but not exactly on, a straight line. Predictions are made by drawing a straight line that fits the data points as closely as possible and by using the line to predict additional data points. This line is called a **best-fit line**, shown in Figure 2.

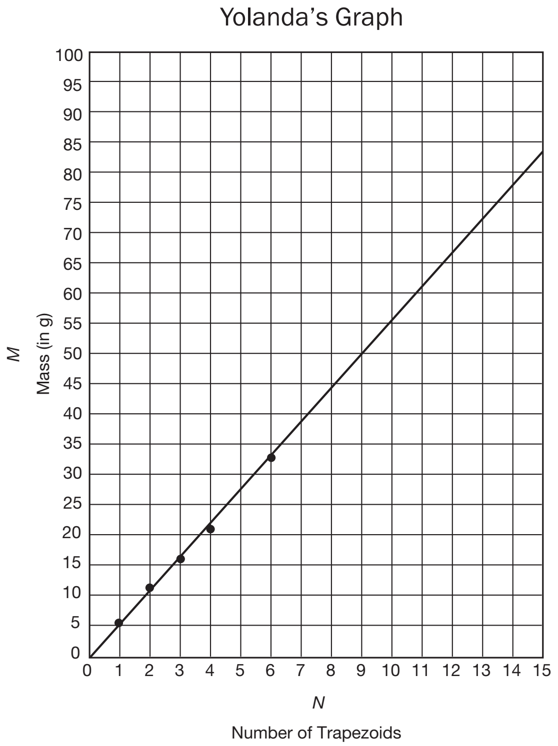

Figure 2: *A graph with a best-fit line*

Measuring Mass

In *Math Trailblazers*™, the exploration of weight and mass begins in the second grade as students use the two-pan balance to compare the masses of various objects. Those experiences help students discover that larger objects are not necessarily heavier. In this unit we will be assigning a number to the mass of an object. As in any measurement situation, we have to decide on a unit of measure. Here, we use the metric unit for mass, the gram. To find the mass of an object, we place it in one of the balance's pans and then put standard masses in the other pan until the two pans balance. See Figure 3. To determine the mass of the object, the standard masses must be added together. Determining the total value of the masses can be done more efficiently with multiplication, providing a nice context for problem solving with multiplication.

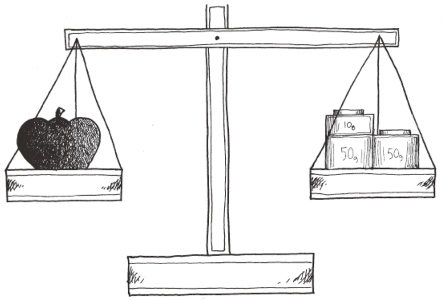

Figure 3: *Determining mass with a two-pan balance and standard masses*

Mass vs. Weight. In everyday language we talk about the weight of an object rather than its mass. Technically, weight and mass are distinct concepts. In scientific terms the **mass** of an object is the amount of matter in the object. Mass is measured in kilograms and grams in the metric system and in pounds and ounces in the English system. An object's weight is the measure of the pull of gravity on that object. It is not important to make a fuss about this terminology in third grade. Children and adults, in their everyday language, usually talk about one object "weighing" more than another. During this unit you will probably hear students saying that they are "weighing" an object, rather than "massing" it. Although we use the correct terms in our materials and prefer that students say something "has a mass of 110 grams" rather than "weighs 110 grams," do not let the informal usage of the word *weigh* disturb you.

There are two pedagogical approaches you can take with regard to the distinction between mass and weight.

1. Ignore it. This point is fairly subtle and eludes many adults.
2. Provide a simple explanation, but don't worry about it.

Because of the awareness of space travel, most children know that the pull of gravity is different on different planets and that there is essentially no gravity in outer space. Many museums and planetariums have an exhibit that shows your weight on the moon and various planets. For example, since the moon's gravity is weaker than the Earth's, the pull on an individual object would be less. Thus, a human being would weigh less on the moon than on Earth.

How is the mass of an object affected by gravity? The mass of an object remains constant regardless of space travel because gravity does not influence the amount of matter in an object. Since we use a two-pan balance to measure mass, both sides of the balance are equally affected by gravity. If an 11-gram pencil balances one 1-gram and two 5-gram standard masses on Earth, it will balance those same masses on the moon. The amount of matter in the pencil and the amount of matter in the standard masses remain the same in both locations.

Measurement Error

For most students, the phrase *measurement error* conjures up images of making a mistake when measuring. This is not the meaning of the phrase. **Measurement error** actually refers to the many reasons a measurement cannot be exact and to the amount of precision that can be expected in a measurement.

There are several reasons a measurement may not be exact. One reason is that there are limitations on the accuracy of any measurement instrument. For example, if someone says she is 5 feet 8 inches tall, she does not mean that she is exactly that height. Rather, when her height was measured, the closest inch mark was the one at 5 feet and 8 inches. If the measuring device used had finer divisions, she might have found her actual height to be 5 feet $8\frac{1}{4}$ inches or 5 feet $7\frac{5}{8}$ inches. The problem associated with measuring mass with a two-pan balance is similar. The smallest standard mass is 1 gram. Therefore, the most precise direct measurement we can make is accurate only to within 1 gram. A second kind of measurement error is due to variations in the object being measured. For example, a person's height varies slightly according to current posture and time of day. (We are usually slightly taller in the morning.) This is another reason people's heights are generally measured only to the nearest inch.

The precision with which a measurement is made often depends on the reason we are making the measurement. For example, if a decorator needs to know the size of a table to buy a tablecloth, he would be interested only in length and width measurements to the nearest inch. If a mover were to move the table through a doorway or if a carpenter were to cover a floor with tile, she might want to know the length and width to the nearest quarter or eighth of an inch.

Math Facts and Mental Math

Multiplication Facts. This unit continues a systematic review and assessment of the student's fluency with the multiplication facts. In this unit students will have opportunities to gain fluency with the multiplication facts for the last six facts (4×6, 4×7, 4×8, 6×7, 6×8, 7×8). Students should be encouraged to reason from facts they know and to break the factors into factors they know. For example, to solve 4×6, use $2 \times 6 = 12$ and $12 + 12 = 24$, so $4 \times 6 = 24$.

Resources

- Carraher, D.W. and A.D. Schliemann "Early Algebra." F.K. Lester, Jr., ed. *Second Handbook of Research on Mathematics Teaching and Learning.* Information Age Publishing, Inc., Charlotte, NC, 2007.
- Kaput, J.J., D.W. Carraher, and M.L. Blanton (eds). *Algebra in the Early Grades*. Lawrence Erlbaum Associates, New York, NY, 2008.
- National Research Council. "Conclusions and Recommendations," in *Adding It Up: Helping Children Learn Mathematics.* J. Kilpatrick, J. Swafford, and B. Findell, eds. National Academy Press, Washington, DC, 2001.
- *Principles and Standards for School Mathematics*. National Council of Teachers of Mathematics, Reston, VA, 2000.