Unit 13

MATHEMATICS IN THIS UNIT
Multiplication, Division, and Volume

Taken from the *Math Trailblazers* digital Teacher Guide

Multiplication and Division Computation

Distributive Law of Multiplication over Addition. In Lesson 1, *Break-Apart Products with Larger Numbers,* students review solving multiplication problems by breaking products into the sum of simpler products. For example, 7×4 can be broken into $5 \times 4 + 2 \times 4$. The fundamental property that is involved in this process is the **distributive property of multiplication over addition** (although students do not study this formally). This property states that for any numbers *a, b,* and *c,*

$$(a + b) \times c = (a \times c) + (b \times c).$$

To find 7×4 as expressed above, we partition 7 into two parts, $7 = 5 + 2$, and apply the distributive property with $a = 5$, $b = 2$, and $c = 4$.

The unit does not include formal instruction on the distributive property. However, the activities in this unit will help students to develop an informal understanding of this property. This type of understanding is very important as it enables students to do many calculations mentally and is the basis for paper-and-pencil multiplication algorithms.

Division with and without Remainders. This unit introduces no formal algorithm for division. Students are encouraged to use a variety of strategies for solving division problems with and without remainders. Strategies such as repeated subtraction, manipulative models, and calculators are emphasized, but you should allow students to explore others. Students learn paper-and-pencil methods for division in fourth and fifth grades.

Types of Computing. Students should become proficient in all three types of computing: mental, calculator, and paper and pencil. They also should develop a sense of when each method is appropriate. Furthermore, they should be able to make estimations as well as use each method to find exact answers. This unit develops a paper-and-pencil algorithm for multiplication, but students can also practice other types of computing. They can use mental math to make estimates to verify the reasonableness of their answers, and they can use calculators to check their exact answers.

This unit introduces students to the all-partials algorithm for multiplication. This algorithm is different from the compact algorithm in that it allows students to write all partial products. The lesson guide for Lesson 2 *More Multiplication Stories* discusses this algorithm. Though we focus on the all-partials algorithm, stu-dents should not be discouraged from using other cor-rect mental methods or paper-and-pencil algorithms that make sense to them.

Volume

In the activities and lab in this unit, students use graduated cylinders and water to measure the volume of solid objects and of containers. In the lab *Fill It Up* students measure the volume of containers by pouring graduated cylinders of water into the containers and using arithmetic operations to determine the total volume of the container. This procedure will be new for most students. Then in *Measuring Volume of Solid Objects,* they find the volume of small solid objects by submerging them in the water in a graduated cylinder and measuring the amount of water that is displaced. This procedure, called **measuring volume by displacement,** will be familiar to students who used the *Math Trailblazers*® curriculum in second grade.

The **volume** of a solid object, such as a rock, is the amount of space the object occupies. The **volume** of a container, such as a bottle or a jar, is the amount of space inside. The latter concept of volume is sometimes referred to as **capacity.** Usually we will use the term *volume* for both concepts since the intended meaning is clear from the context.

In this unit, students progress from using a centimeter as a unit of length and a square centimeter as a unit of area to using a cubic centimeter as a unit of volume. A **cubic centimeter (cc)** is the volume of a cube whose edges each have a length of 1 centimeter.

Students may be familiar with a larger metric unit of volume, the liter, since many beverages are sold in 1- and 2-liter bottles. A **liter (l)** is 1000 cubic centimeters. Another unit of volume frequently seen on graduated cylinders is the **milliliter (ml).** Since "milli" is a prefix that means "thousandth," a liter is also 1000 milliliters. Therefore, converting between milliliters and cubic centimeters is simple:

1 milliliter = 1 cubic centimeter

1 ml = 1 cc

Math Facts and Mental Math

Multiplication Facts. This unit culminates the systematic review and assessment of the students' fluency with the multiplication facts. In this unit students will have opportunities to gain fluency with the multiplication facts for the last six facts (4×6, 4×7, 4×8, 6×7, 6×8, 7×8). Students should be encouraged to reason from facts they know and to break the factors into factors they know. For example, to solve 4×6 use $2 \times 6 = 12$ and $12 + 12 = 24$ so $4 \times 6 = 24$. Students will also be assessed on their fluency with all the multiplication facts.

Resources

- Carraher, D.W. and A.D. Schliemann. "Early Algebra." *Second Handbook of Research on Mathematics Teaching and Learning.* F.K. Lester, Jr., ed. Information Age Publishing, Inc., Charlotte, NC, 2007.
- Fuson, K.C. "Developing Mathematical Power in Whole Number Operations." In *A Research Companion to Principles and Standards for School Mathematics.* J. Kilpatrick, W.G. Martin, and D. Schifter, eds. National Council of Teachers of Mathematics, Reston, VA, 2003.
- Kaput, J.J. *Algebra in the Early Grades*. Lawrence Erlbaum Associates, New York, NY, 2008.
- Lampert, M. "Teaching Multiplication." *Journal of Mathematical Behavior.* 5, 241–380, Norwood, NJ, 1986.
- National Research Council. "Conclusions and Recommendations," in *Adding It Up: Helping Children Learn Mathematics.* J. Kilpatrick, J. Swafford, and B. Findell, eds. National Academy Press, Washington, DC, 2001.
- National Research Council. "Developing Proficiency with Whole Numbers." In *Adding It Up: Helping Children Learn Mathematics.* J. Kilpatrick, J. Swafford, and B. Findell, eds. National Academy Press, Washington, DC, 2001.
- *Principles and Standards for School Mathematics.* National Council of Teachers of Mathematics, Reston, VA, 2000.
- Van de Walle, J.A., K.S. Karp, and J.M. Bay-Williams. *Elementary and Middle School Mathematics: Teaching Developmentally, Eighth Edition.* Pearson Education, Inc., Upper Saddle River, NJ, 2013.