Unit 1

MATHEMATICS IN THIS UNIT
Data About Us

Taken from the *Math Trailblazers* digital Teacher Guide

In this unit, students use graphs and tables to model the relationship between categorical and numerical variables. Along with bar graphs and point graphs, students look to measures of central tendency (mean and mode) to make generalizations and predictions about the population.

In the first three lessons, students are introduced to a fictional classroom of students. Students in this Chicago classroom collect and organize data about each other in order to describe themselves to their pen pals in Arizona. Within this context, students learn how to use variables and averages (medians) as they solve problems and review methods of collecting and representing data. In this process, students also design survey questions and collect, represent, and analyze the resulting data.

TIMS Laboratory Method. This unit reviews the four steps of the TIMS Laboratory Method: drawing a picture, collecting the data, graphing the data, and analyzing the results.

The four steps of the TIMS Laboratory Method will be familiar to students who have used *Math Trailblazers* before. Others will become accustomed to the routines as they gain experience with the method. Although we want students eventually to apply this method on their own, many may need to be guided at first. One of your more difficult instructional decisions will be how much guidance to give—how to balance imitation and autonomy. Learning by imitation will make for more orderly lessons, not an unimportant consideration in the beginning of the year. On the other hand, too much imitation can undermine student autonomy and can foster misconceptions about what mathematics and science are. As the year progresses, you will want to see students doing more work on their own, figuring out their strategies and solving problems with little teacher guidance.

Represent Data. In this unit, students will make and interpret bar graphs and point graphs. Since the degree of familiarity with graphing and the TIMS Laboratory Method will vary from class to class, we offer a range of approaches in the first lab, *Arm Spanvs. Height* (Lesson 5). We build flexibility into the lab so that you can adapt it to the needs of your class-room while you are getting to know your students. One aspect of the method to stress in these early experiments is how the real objects—the picture, the data table(s), and the graph(s)—all represent the same situation. Many students benefit from discus-sions connecting and comparing these various repre-sentations.

Students also learn to work with values written as **ordered pairs**, e.g., (12, 5). In this convention, the value measured on the horizontal axis is written first and the value measured on the vertical axis is written second.

Variables and Values. In this first lab, your students will investigate the relationship between two variables, the arm span and the height of their classmates. Variables are an important part of both mathematics and science.

Several basic procedures for handling variables are involved in this unit: distinguishing between variables and values, denoting variables by symbols, and labeling data table columns and graph axes with the names of variables. Students learn to use more specific terminology for variables—numerical and categorical variables. Variables that have numbers as their values are **numerical;** variables that have nonnumerical values are **categorical.** For example, number of pets and height are numerical variables because possible values for these variables (1 pet or 2 pets and 54” or 60”) are quantities. Categorical variables studied in this unit include eye color and favorite sport because possible values for these variables (black or brown and soccer or football) have to do with qualities, not quantities.

Stress that an experiment is an investigation about relationships between variables. For example, in the experiment *Arm Span vs. Height,* students will determine whether they can predict a fourth-grader’s height given his or her arm span.

Algebra in the Early Grades

There is a debate among mathematicians and educators about the content of algebra and especially the content of algebra in the early grades. Some have defined algebra as modeling, as pattern finding, as the study of structure, and as making sense of one's world quantitatively.

> "We advocate for an early emphasis on developing children's ability to conceive of, reason about, and manipulate complex ideas and relationships, as an equal complement to numerical reasoning and computation. Children who develop a rich capacity for reasoning about general relationships among quantities will possess the conceptual foundation for learning and making sense of different programs and views of algebra."
>
> (Smith and Thompson, 2008)

This unit draws heavily on the connection between patterns and data. Through the use of tables, graphs, variables, and the TIMS Laboratory Method, students not only find ways to organize numerical and categorical data, they also discover patterns and relationships in those data. This focus on patterns captures the most central idea of early algebraic reasoning. The activities in this unit demonstrate the connections between the data strand and the algebra strand in *Math Trailblazers*.

This focus on early algebraic thinking is developed throughout *Math Trailblazers.* Following recommendations of researchers, activities are integrated through all grades that give students concepts and skills needed to make a natural transition from learning and doing arithmetic to learning and doing algebra. (Carpenter et al, 2003; Carraher and Schliemann, 2007; Kaput, 2008; Kilpatrick and Izsak, 2008; Schliemann et al, 2007). Students extend and connect their concepts of number, geometry, measurement, and data to develop tools for algebraic reasoning. These tools enable them to "do algebra," that is, to identify, describe, visualize, and simplify patterns and relationships. They learn to generalize procedures while they use arithmetic. They also learn to make generalizations about numbers that are collected in data sets, organized in tables, and pictured in graphs.

> "Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved, attending to the meaning of the quantities . . ."
>
> (Common Core State Standards Initiative, 2010)

Review and Practice

Every unit includes opportunities for distributed practice of concepts and skills. These resources can be found primarily in two places:

- Daily Practice and Problems (DPP) in the *Teacher Guide*
- Home Practice in the *Teacher Guide*

Daily Practice and Problems is a set of short exercises that provides ongoing review and study of math concepts and skills and provides a structure for systematically reviewing basic math facts. See the Daily Practice and Problems for this unit for more information about how they are organized and for ways to incorporate DPP items into your daily routine.

The Home Practice section of the *Student Activity Book* is a series of problems that supplements the homework included in the lessons. The Home Practice distributes skills practice throughout the units and reviews concepts studied in previous units.

Math Facts and Mental Math

Addition Facts. In this unit, students review and are assessed on their addition facts. Students have been developing strategies for solving the addition facts since their early years and should have gained fluency. See the Letter Home for more about how the facts are grouped by strategy. The subtraction facts are reviewed in Unit 2, and a systematic review of the multiplication and division facts begins in Unit 3.

Resources

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