

Place Value and Number Sense. A child's understanding of numbers occurs gradually. Children need many opportunities to work with numbers in real-world contexts in order to develop an intuition about numbers—what is called number sense. They also need an understanding of place value to be able to compute accurately and efficiently with large numbers. It is necessary to develop these basic number concepts as a foundation of understanding for computational procedures.

This unit provides many opportunities for students to explore number meaning and to develop number sense. Students collect large numbers from a variety of print material and data bases. They then explore place value as they practice reading and ordering these numbers. They represent larger numbers using base-ten hoppers on number lines and place value charts. They extend the base-ten pieces for 1; 10; 100; and 1000 by building physical models of 10,000; 100,000; and 1,000,000. This provides an opportunity for students to see the relative size of numbers as they explore the patterns in our place value system. Students gain experience using numbers written as symbols as they link the numbers to names and physical models.

Estimation and Number Sense. Estimation is a useful tool in our everyday lives that leads to better number sense. Students explore strategies for computational estimation in this unit. Students who are able to estimate well tend to be more flexible in their thinking, use a variety of estimation strategies, and demonstrate a deeper understanding of numbers and operations.

While there are times when exact answers are necessary in the computation of large numbers, an estimate is often all that is needed or possible.

Rounding numbers is introduced as one strategy for computational estimation. Using round numbers allows us to compute mentally. The emphasis in this unit is not on the rules for rounding numbers but rather on the context in which round numbers are used. Reliance on a rule may lead to confusion and solutions which are not reasonable. Students need to pay attention to the value of the numbers in each context. One of the reasons we use round numbers is to estimate approximate answers to addition, subtraction, multiplication, and division problems so we can compute more easily in our heads.

Students need the opportunity to discuss when estimation is appropriate and when exact answers are needed. This unit provides activities in which this discussion can occur.

Math Facts and Mental Math

Multiplication Facts. This unit marks the end of students' review of the multiplication facts in Units 3–5. In each unit, students reviewed strategies for learning the facts in small groups, practiced the facts in the Daily Practice and Problems (DPP), and then took a short quiz on the fact families and the multiplication facts. Students track their own progress by circling the facts they know on their *Multiplication Facts I Know* charts.

This unit includes a *Multiplication Facts Inventory Test* as part of the DPP items in Lesson 8. We include this test at this point in the curriculum based on recommendations from research. In reference to timed tests, the National Research Council states:

This scattershot form of practice is, in our opinion, rarely the best use of practice time. Early in learning it can be discouraging for students who have learned only primitive, inefficient procedures. The experience can adversely affect students' disposition toward mathematics, especially if the tests are used to compare their performance. If appropriately delayed, timed tests can benefit some students, but targeted forms of practice, particularly combinations that have yet to be mastered or on which efficient procedures can be used are usually more effective (NRC, 2001).

How to Use the *Multiplication Facts Inventory Test*. Lesson 1 asks students to review their *Multiplication Facts I Know* charts and to make a list of the facts that are not circled, so they can study them for the Inventory Test using Triangle Flash Cards. The *Student Activity Book* includes blank Triangle Flash Cards, so students can make new cards for the facts that they still need to learn. Encourage students to take the flash cards home and focus on learning those facts. As part of the class discussion in Lesson 1, discuss strategies for those facts that several students still need to learn. These are often facts in the Last Six Facts group (4×6 , 4×7 , 4×8 , 6×7 , 6×8 , 7×8) which lend themselves to doubling or break-apart strategies.

After students have taken the Inventory Test, ask each student to update their *Multiplication Facts I Know* charts and make a new list of any facts they still need to study. In line with research recommendations, focus students' practice just on these facts.

How Not to Use the *Multiplication Facts Inventory Test*. An old saying that applies to testing students' knowledge of the math facts goes, "You can't fatten a lamb by weighing it." It does little good—and as noted above can do harm—to repeatedly ask all students to complete timed tests of 100 facts or more. Such "repeated weighings" do not efficiently support students' learning of math facts. Instead of using the Inventory Test or similar tests in this way, use the test as a diagnostic tool and develop "targeted forms of practice" for those facts students are still having difficulty with. Encourage students to develop efficient strategies for these facts, as opposed to just memorization.

Division Facts. This unit begins the systematic study of the division facts in Grade 4. Our goal is for students to develop fluency with the division facts by the end of the year. In Lesson 1, students learn to quiz themselves using Triangle Flash Cards and how to track their learning using *Division Facts I Know* charts. They begin with the 5s and 10s.

The goal of the math facts development in *Math Trailblazers* is for students to learn the basic facts efficiently, gain fluency with their use, and retain that fluency over time. A large body of research supports

an approach in which students develop strategies for figuring out the facts rather than relying on rote memorization. This not only leads to more effective learning and better retention, but also to the development of mental math skills. In fact, too much drill before conceptual understanding may interfere with a child's ability to understand concepts at a later date.

The research that contributed to the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* and the National Research Council's *Adding It Up* has informed the structure of the math facts strand in *Math Trailblazers*. Also see the Unit 6 Letter Home for more about the development of the basic facts in 4th-Grade *Math Trailblazers*.

The main strategy for learning the division facts is to use the related multiplication facts. Therefore, expectations for fluency have been delayed until students have reasonable fluency with the multiplication facts. In Units 3–5, they used fact families in the DPP to connect the multiplication facts to the division facts. This use of fact families is continued in the upcoming units. See Figure 1. Each unit has two Expectations related to the math facts—one for the assigned group of division facts and one for fact families. See the *Key Assessment Opportunities Chart* following the Assessment In This Unit section.

Unit	Division Facts Groups	Focus
6	5s and 10s	Use strategies fluently
7	2s and 3s	
8	9s	
9	Square Numbers	
10	Last Six Facts	
11	Last Six Facts	
12	Review all division facts	

Figure 1: Development of division math facts in Grade 4

Division and Zero

Division problems involving zero can be a source of confusion for students. One way to think about division involving zero is to consider the corresponding multiplication problem. For example, to solve $0 \div 4 = n$, we must find a number n that makes $n \times 4 = 0$ true. Since $0 \times 4 = 0$, then $n = 0$ and $0 \div 4 = 0$. Since any number times zero is zero, zero divided by any number (other than zero) is zero.

To solve $4 \div 0 = n$, we must find a number n that makes $n \times 0 = 4$ a true statement. Since any number times zero is zero, there is no number for n that will make the statement true. So, we say that division by zero is **undefined**.

Consider $0 \div 0 = n$. What number n will make the statement $n \times 0 = 0$ true? $1 \times 0 = 0$, $2 \times 0 = 0$, $3 \times 0 = 0$, and so on. In fact, any number will make the statement true. Since there is no unique solution to $0 \div 0 = n$, we say that $0 \div 0$ is undefined.

Resources

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