MATHEMATICS IN THIS UNIT
Angles, Lines, and Shapes

Taken from the *Math Trailblazers* digital Teacher Guide

"Given children's affinity toward, knowledge of, and ability to gain geometric knowledge, it is important that this domain of mathematics not be neglected. Instruction in geometry needs to complement the study of number and operation in grades pre-K to 8" (National Research Council, 2001). *Math Trailblazers* echoes this expectation by emphasizing the importance of geometry in the mathematics curriculum. This unit represents the focal point of geometry in fourth grade. Students explore the nature of two-dimensional geometric elements including lines, angles, and polygons. Students also discover relationships within and among these elements as they advance their understanding through stages, from basic intuition to analysis and informal deduction.

Van Hiele Levels of Geometric Development

Much of the approach to geometry found in the *Math Trailblazers* curriculum is grounded in the insights of Dutch educators Pierre van Hiele and Dina van Hiele-Geldof. Ongoing research in mathematics education continues to confirm the five levels of geometric development first described by the van Hieles in the 1950s (Burger and Shaughnessy, 1986). The five levels are:

Level 0: Visualization. Students judge geometric objects by their appearance, but not by attributes. For example, a student can identify a rectangle because it "looks like a rectangle," but not because it has opposite sides equal and four right angles.

Level 1: Analysis. Students begin to describe the properties of objects. A figure is no longer judged because it "looks like one," but rather because it has certain properties. For example, an equilateral triangle has three equal sides, three equal angles, and line symmetry.

Level 2: Informal Deduction. Students logically order the properties of figures and are able to deduce that one property precedes or follows from another property. They see relationships among figures. For example, a square has all the properties of a rectangle; therefore, a square is a rectangle. Students may also be able to define a square based on its properties.

Level 3: Deduction. Students write formal proofs based in an axiomatic system. A rigorous high school geometry course is taught at level 3.

Level 4: Rigor. Students can work with different axiomatic systems. This level corresponds to college work in geometry (Crowley, 1987; van Hiele, 1999).

The van Hieles found that each level, while not age specific, builds on the previous level. Students proceed from level to level sequentially and no level can be omitted. Advancement depends on content and method of instruction (Crowley, 1987; van Hiele, 1999). Moreover, a student's experiences with lower-level reasoning at the elementary school level are critical to success with geometry in later schooling. Students who are at level 0 or 1 when entering high school geometry have a poor chance of success. Students who begin high school geometry at level 2 have at least a 50% chance at succeeding (Senk, 1989). Unfortunately, many upper elementary students are still at level 0. This is not surprising, as researchers have found that most geometry questions asked in standard elementary math textbooks were answerable with level 0 understanding (Fuys, Geddes, and Tischler, 1988).

In this unit, we start with level 0 ideas by asking students to identify and draw various geometric figures. Most of the work in this unit is at level 1, where students describe the properties of lines, angles, and shapes. Level 2 ideas are introduced as students explore ways to classify these figures.

The content of this unit focuses on figures and shapes in two dimensions, beginning with some of the basic elements of two-dimensional geometry—lines and angles. Later in the unit, students work with polygons to analyze their properties, relationships, transformations, and classifications.

Lines. The concept of line is fundamental to geometry. When we say "**line**" in mathematics, we always mean a "straight line" that extends forever in both directions. Although there are formal ways to define a line mathematically, at this stage students use intuitive notions that support their emerging geometrical reasoning. Since the edge of a ruler is straight, a figure carefully drawn alongside the edge of the ruler will represent a part of a line, or line segment. Another way students can represent a line segment is to fold a piece of paper.

Students also study the relationships between lines. The ways that lines intersect—or do not intersect—in a plane determines whether they are parallel, perpendicular, or neither.

Angles. Children first understand angles intuitively as corners in shapes, crossings and bends of lines, and rays turning about a pivot (Clemens & Sarama, 2009). Only after these intuitive ideas are established do students move toward more quantitative understandings about angles' sizes and measures. As a starting point in this unit, students begin exploring and comparing angles using intuitive models such as bent straws and corners of plastic shapes. They then progress to angle classification based on comparisons to right angles. Later in the unit, students use a protractor to measure angle size to the nearest degree.

Children often have misconceptions about angles. For example, students often believe that the size of an angle depends on the length of the sides of the angle (Wilson and Osborne, 1988), or that right angles are angles that open to the right (Crowley, 1987). To avoid such misconceptions, children must see angles drawn in different positions and with sides of different lengths.

Symmetry. In relation to van Hiele's levels of geometric thinking, initial (level-0) ideas about symmetry are visual and intuitive. As these ideas develop further, students begin to understand symmetry as a property to describe and analyze shapes (Crowley, 1987). There are three types of symmetry in the plane: 1) line (reflective) symmetry, 2) turn (rotational) symmetry, and 3) translational symmetry. See Figures 1, 2, and 3, respectively.

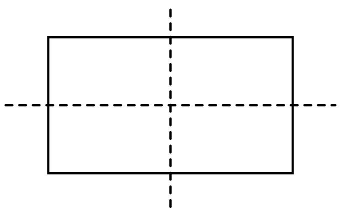

Figure 1: *A rectangle with two lines of symmetry*

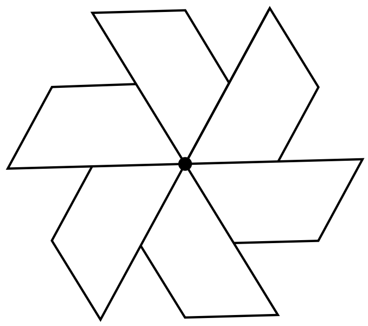

Figure 2: *A shape with $\frac{1}{6}$-turn symmetry*

Figure 3: *A shape with translational symmetry*

Shapes that have **line symmetry** coincide with themselves when reflected over a line, or axis of symmetry. Students are likely most familiar with this type of symmetry found in rectangles, an uppercase letter "A", or in the shape of a butterfly. A figure that has line symmetry has at least one line of symmetry. If such a figure is drawn on paper and folded along this line, the two pieces will match exactly. Some figures have more than one line of symmetry, as shown in Figure 1.

Shapes with turn, or rotational, symmetry coincide with themselves when turned about a point called the center of rotation. A shape has **turn symmetry** if it can be turned less than 360° and fit on itself exactly. For example, the shape in Figure 2 repeats itself six times before making a full turn about its center of rotation, so we say this figure has $\frac{1}{6}$-turn symmetry. Other authors refer to this as sixfold symmetry. Often, figures that have turn symmetry have line symmetry as well. For example, the shape in Figure 1 has two lines of symmetry and $\frac{1}{2}$-turn symmetry. The third type of symmetry is called translational.

A figure with **translational symmetry** is repeated over and over by sliding it a fixed distance, as shown in Figure 3. This unit focuses on line and rotational symmetry.

The study of symmetry gives names to patterns children see in everyday life and in art. For example, we can see patterns of symmetry in architecture as well as in the petals of many flowers. Students who have been introduced to these notions will be better able to analyze and describe what they see.

Motion and Congruence. An understanding of motion is central to the early learning of geometric concepts. Lines are understood as *extending* in two directions. Angles are modeled as *opening, bending,* or *turning* through a rotation. In this unit, motion is further developed as applying slides, flips, and turns to shapes. These types of motion in geometry are called rigid transformations because they do not change the form or outline of the shape, only its location and orientation. Students apply slides, flips, and turns to shapes and observe the effects of these movements. They observe repeating quilt patterns and manipulate polygons to identify how a shape transforms from one position to another.

In studying the effects of slides, flips, and turns on shapes, students discover that these transformations do not alter the shape itself. This understanding leads to the concept of congruence. Students discover that one shape is **congruent** to another when the second shape can be made by any combination of slides, flips, and turns done on the first shape. This understanding lays the conceptual foundation for a more formal study of congruence in later grades.

Analyzing Shapes. As students build their understanding about lines, angles, and symmetry, they use these ideas to study shapes geometrically. Power Polygons™ provide an opportunity to investigate the properties of shapes by manipulating, combining, and overlaying polygons. Starting from visual, level-0 ideas about shapes, students progress into level-1 reasoning by identifying properties of various polygons, including special triangles and quadrilaterals. Triangles are analyzed and classified by the types of angles they contain. Students also discover that the sum of interior angles of a triangle is 180 degrees.

Toward the end of the unit, students look for properties of various types of quadrilaterals, including trapezoids, parallelograms, rhombuses, rectangles, and squares. Properties such as line symmetry, parallel sides, perpendicularity, and equal-length sides are investigated to describe and sort shapes. Finally, a more deductive (level-2) approach is introduced where students use inclusion to reason about how polygons are classified. For example, since a square is a quadrilateral with four right angles, a square is also a rectangle. It may be that some of your students will not comprehend the concept of inclusion by the end of this unit. Exposing them to the ideas of inclusion will help them develop understanding gradually, though mastery is not required at this time.

A Word on Vocabulary

This unit contains a great deal of mathematical vocabulary. Encourage students to use mathematical words that precisely and accurately express their thoughts. However, keep in mind that students remember definitions with greater meaning when they learn them through discourse. Direct memorization is generally far less productive than using words in context, since words learned by rote are quickly forgotten. A better strategy is to encourage students to discuss their work using their language and then model geometers' terminology to extend the discussion.

To help students with vocabulary acquisition, a Geometry Word Chart is introduced as a way to summarize and organize terms. New words are added to the chart as they are introduced. Students can look at the chart during the unit to revisit words and examples as they come up again in context. Students also add and refine their representations on the chart as their understanding of concepts grows.

Etymology, or the study of words, teaches us that words often do not have an exact, single meaning. Word meanings evolve over time, and words may be defined differently among different groups or individuals. This is even true in mathematics. For example, in a study of school geometry texts, the word "quadrilateral" was found to have seven different definitions (Usiskin, 2008). Although each of these definitions includes the same set of shapes, each emphasizes something different about quadrilaterals. When considering the word "trapezoid," we even find two definitions in textbooks that include *different* sets of shapes. One definition includes quadrilaterals with *only* one pair of parallel sides; another with *at least* one pair of parallel sides. In the end, the class must agree to settle on a particular definition, but we should do so knowing mathematicians sometime disagree.

Math Facts and Mental Math

This unit continues the review and assessment of the division facts to develop mental math strategies, gain proficiency, and to learn to apply multiplication strategies to larger numbers. Students will focus on the division facts for the square numbers.

Resources

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