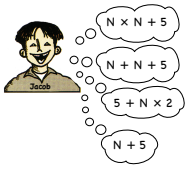
Patterns and Problems

1. Jacob's function table is missing its rule. Help Jacob find the rule for his function table. Which of Jacob's rules do you agree with? Show or tell how you know your rule works.

Jacob's Table

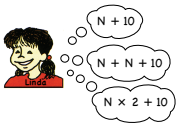
Input	Output
0	5
1	7
2	9
3	11
4	13
5	15
N	



2. Linda's function table is also missing its rule. Help Linda find the rule for her function table. Which of Linda's rules do you agree with? Show or tell how you know your rule works.

Linda's Table

Input	Output
0	10
1	12
2	14
3	16
4	18
5	20
N	



3. Look at Linda's and Jacob's rules in Questions 2 and 3.
   - **A.** How are they alike?
   - **B.** How are they different?

Use the *Rules, Tables, and Graphs* pages in your *Student Activity Book* to review patterns and functions.

***Student Guide* - Page 577**

Name _____ Date _____

Rules, Tables, and Graphs

1. Ming's function machine triples a number, then subtracts three. Jackie's function machine doubles a number, then subtracts two.

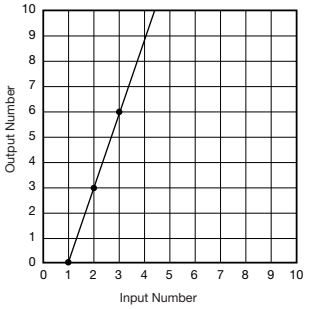
Table A

Input	Output	Ordered Pairs (Input, Output)
1	0	(1, 0)
2	3	(2, 3)
3	6	(3, 6)

Table B

Input	Output	Ordered Pairs (Input, Output)
1	0	(1, 0)
2	2	(2, 2)
3	4	(3, 4)

- **A.** Which function table is Ming's? _____
- **B.** Which function table is Jackie's? _____
- **C.** Ming and Jackie graphed the data in their function tables. Is this Ming's graph or Jackie's graph? How do you know?



- **D.** Graph the points in the other table. Draw a line.

***Student Activity Book*- Page 565**

Student Guide

Questions 1–3 (SG pp. 577)

1.* N + N + 5 and 5 + N × 2. Possible response: both N + N + 5 and 5 + N × 2 work. I tried each for all the inputs and they work. I know N + 5 does not work because 1 + 5 does not equal 7. N × N + 5 did not work either.
3 × 3 + 5 = 14, not 11.

2.* N + N + 10 and N × 2 + 10. Possible response: I tried them both and they worked for every input in the table. I know N + 10 does not work because 2 + 10 does not equal 14.

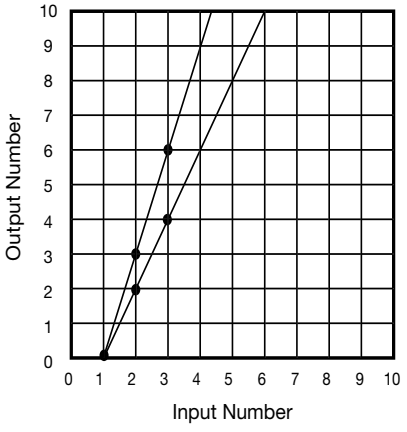
3. **A.*** Linda and Jacob both double N.
   B.* Linda adds 10. Jacob adds 5.

Student Activity Book

Rules, Tables, and Graphs

Questions 1–10 (SAB pp. 565–570)

1. **A.** Table A is Ming's.
   B. Table B is Jackie's.
   C.* Ming. Possible responses: I matched up the data in the table with the data points on the graph; or I noticed the "step" in the graph was 3 and Ming's table also has a "step" of three between points.
   D.*



*Answers and/or discussion are included in the lesson.

2. A.* Rule: Double Plus Two

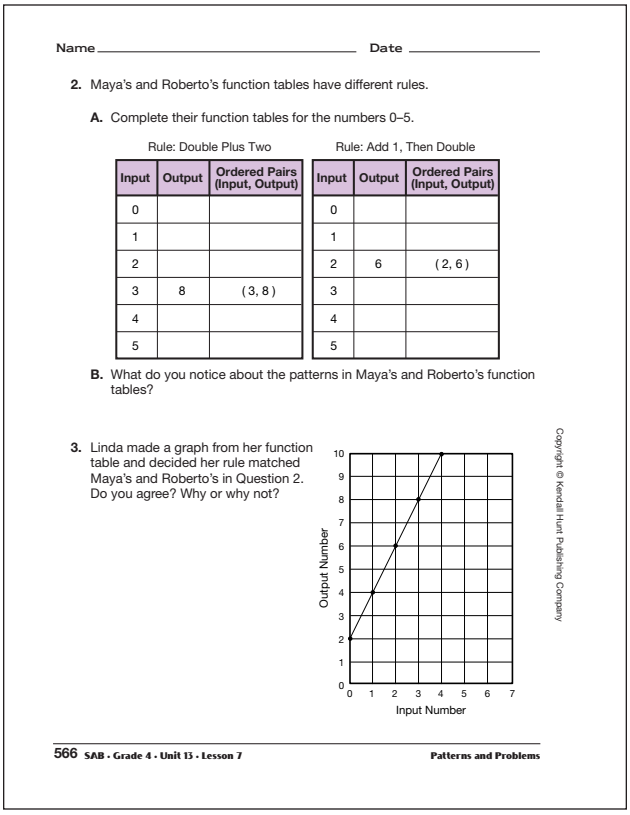
Input	Output	Ordered Pairs (Input, Output)
0	2	(0, 2)
1	4	(1, 4)
2	6	(2, 6)
3	8	(3, 8)
4	10	(4, 10)
5	12	(5, 12)

Rule: Add 1, Then Double

Input	Output	Ordered Pairs (Input, Output)
0	2	(0, 2)
1	4	(1, 4)
2	6	(2, 6)
3	8	(3, 8)
4	10	(4, 10)
5	12	(5, 12)

B.* The inputs and outputs are the same in each table. Doubling and adding two is the same as adding one then doubling.

3. Yes, I agree with Linda. The data in the table matches the data in the graph. The points are the same as the ordered pairs.



Name _____ Date _____

2. Maya's and Roberto's function tables have different rules.

A. Complete their function tables for the numbers 0–5.

Rule: Double Plus Two

Input	Output	Ordered Pairs (Input, Output)
0		
1		
2		
3	8	(3, 8)
4		
5		

Rule: Add 1, Then Double

Input	Output	Ordered Pairs (Input, Output)
0		
1		
2	6	(2, 6)
3		
4		
5		

B. What do you notice about the patterns in Maya's and Roberto's function tables?

3. Linda made a graph from her function table and decided her rule matched Maya's and Roberto's in Question 2. Do you agree? Why or why not?

566 SAB - Grade 4 - Unit 13 - Lesson 7 Patterns and Problems

***Student Activity Book* - Page 566**

***Answers and/or discussion are included in the lesson.**

Name _____ Date _____

4. Nila's sandwich had a mass of 153 grams. She took one bite and then the mass of her sandwich was 128 grams.

A. If each of Nila's bites has the same mass, what is the mass of two bites? Show or tell how you know.

B. What is the mass of three bites?

C. Nila made a table to predict the mass of the sandwich after each bite. Complete the table.

Nila's Sandwich

N Number of Bites	*M* Mass of Sandwich (grams)
0	153
1	128
2	
3	
4	
5	

D. Assuming still that each of her bites has the same mass, predict the Number of Bites *(N)* it will take Nila to eat her whole sandwich. Show or tell how you made your prediction.

E. Which is a rule to find the mass of Nila's sandwich *(M)* if you know the number of bites *(N)* taken? Circle the rule.

M = 128 − 25 × N M = 153 − 25 × N M = 153 − N

***Student Activity Book* - Page 567**

Name _____ Date _____

5. John's sandwich had a mass of 189 grams. After he took one bite, the mass was 159 grams.

A. John made a table to predict the mass of the sandwich after each bite. He used the same mass for each bite. Complete the table.

John's Sandwich

N Number of Bites	*M* Mass of Sandwich (grams)
0	189
1	159
2	
3	
4	
5	

B. Assuming still that each of his bites has the same mass, predict the Number of Bites *(N)* it will take John to eat his whole sandwich. Show or tell how you made your prediction.

C. Write a rule to find the mass of John's sandwich *(M)* if you know the number of bites *(N)*.

6. Use your answers to Questions 4 and 5 to answer these questions:

A. Who has a bigger bite size, Nila or John?

B. How are Nila and John's rules alike?

C. How are they different?

***Student Activity Book* - Page 568**

***Answers and/or discussion are included in the lesson.**

4. **A.** 50 grams;
153 grams – 128 grams = 25 grams in one bite. So two bites is 50 grams.

B. 75 grams

C. Nila's Sandwich

N Number of Bites	*M* Mass of Sandwich (grams)
0	153
1	128
2	103
3	78
4	53
5	28

D. 7 bites, though the seventh bite will be small at about 3 grams. Possible strategy: I added two more rows to the table. At 6 bites there were 3 grams left, so there was one more small bite after that.

E. M = 153 − 25 × N

5. **A.** John's Sandwich

N Number of Bites	*M* Mass of Sandwich (grams)
0	189
1	159
2	129
3	99
4	69
5	39

B. 7 bites. Possible strategy: I subtracted 30 grams from 39 and that was for 6 bites. There were only 9 grams left, so there will be one more small bite for bite number 7.

C.* M = 189 − 30 × N

6. **A.** John

B. For both, the mass of the sandwich is equal to the starting mass minus the mass of a bite times the number of bites.

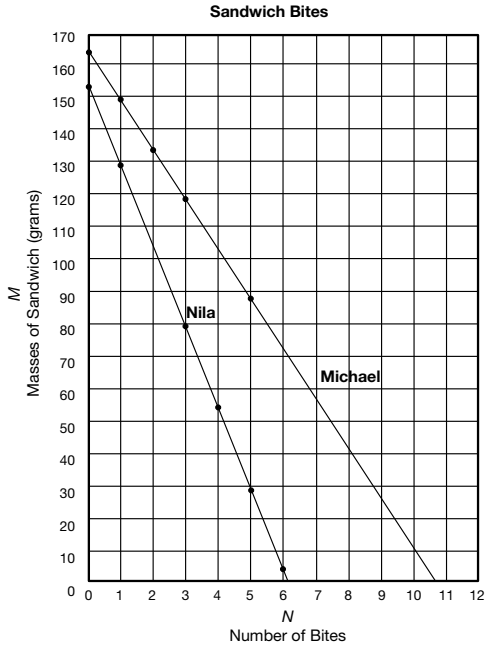
C.* The starting masses are different and the size of the bites is different. Nila's sandwich was smaller than John's. John's bites are bigger than Nila's.

7. A. 15 grams

B. 11 bites

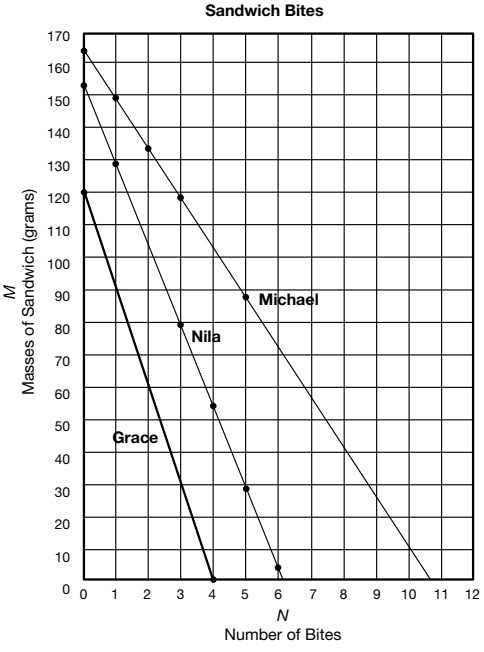
C. Michael has the smallest bite size at 15 grams; Nila's bite size is 25 grams; John's bite size is 30 grams.

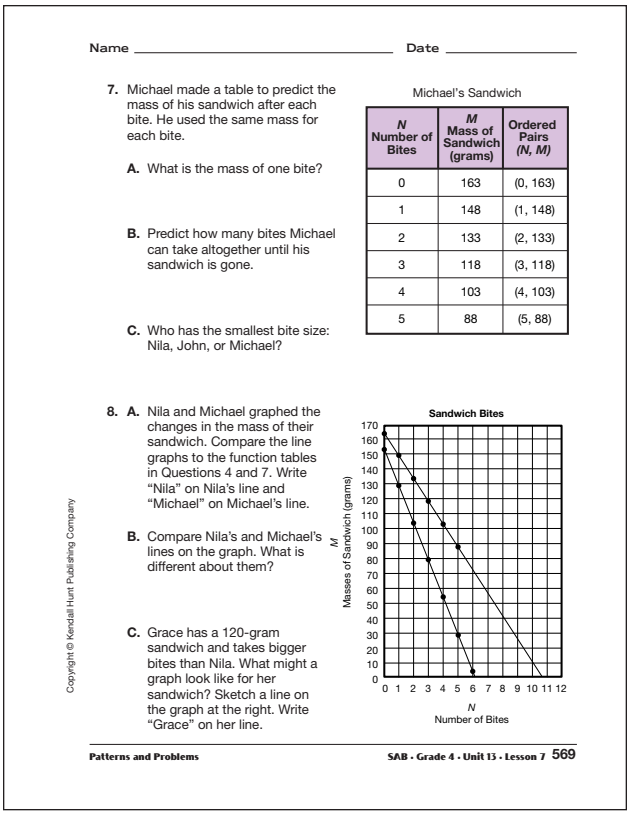
8. A.



B. Possible response: Michael's line is longer. Nila's line starts at (0, 153) and Michael's starts at (0, 163). Michael's line shows that it takes 11 bites to eat the whole sandwich, because when mass (*M*) equals 0, the Number of Bites (*N*) is close to 11.

C.*





Name _____ Date _____

7. Michael made a table to predict the mass of his sandwich after each bite. He used the same mass for each bite.

Michael's Sandwich

N Number of Bites	*M* Mass of Sandwich (grams)	Ordered Pairs *(N, M)*
0	163	(0, 163)
1	148	(1, 148)
2	133	(2, 133)
3	118	(3, 118)
4	103	(4, 103)
5	88	(5, 88)

A. What is the mass of one bite?

B. Predict how many bites Michael can take altogether until his sandwich is gone.

C. Who has the smallest bite size: Nila, John, or Michael?

8. A. Nila and Michael graphed the changes in the mass of their sandwich. Compare the line graphs to the function tables in Questions 4 and 7. Write "Nila" on Nila's line and "Michael" on Michael's line.

B. Compare Nila's and Michael's lines on the graph. What is different about them?

C. Grace has a 120-gram sandwich and takes bigger bites than Nila. What might a graph look like for her sandwich? Sketch a line on the graph at the right. Write "Grace" on her line.

Sandwich Bites
M
Masses of Sandwich (grams)
N
Number of Bites

Patterns and Problems SAB • Grade 4 • Unit 13 • Lesson 7 569

***Student Activity Book* - Page 569**

***Answers and/or discussion are included in the lesson.**

Name _____ Date _____

✓ **Check-In: Questions 9-10**

9. Professor Peabody is planning a 7-day trip. He decides to leave 40 grams of food for his hamster, named Ham. He started this data table.

A. Complete the table.

B. Complete Ham's food graph below using Professor Peabody's data. If the points form a line, draw a best-fit line.

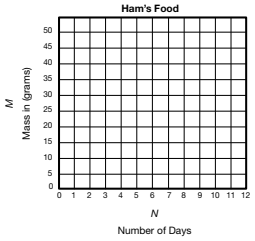
Ham's Food

N Number of Days	*M* Mass (Grams)	Ordered Pairs (*N, M*)
0	40	(0, 40)
1	35	(1, 35)
2	30	(2, 30)
3	25	(3, 25)
4		
5		
6		
7		

10. Use the table or graph to answer these questions.

A. How many grams of food does the hamster eat each day?

B. Write a rule that will tell Professor Peabody the Mass in Grams *(M)* of the food if he knows the Number of Days *(N)*. You may use words or number sentences.



C. Is 40 grams enough food for 7 days? Show or tell how you know.

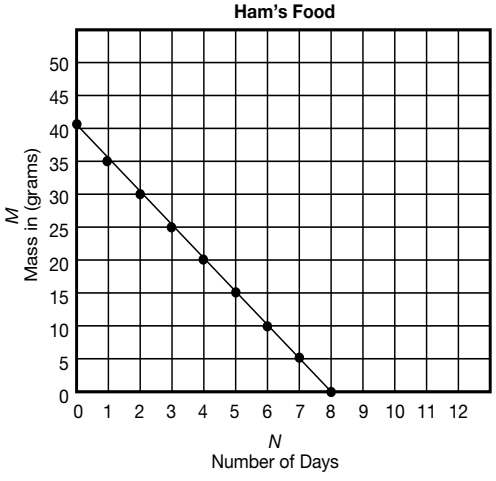
570 SAB • Grade 4 • Unit 13 • Lesson 7 Patterns and Problems

***Student Activity Book* - Page 570**

9. A.

N Number of Days	*M* Mass (grams)	*Ordered* Pairs (N,M)
0	40	(0, 40)
1	35	(1, 35)
2	30	(2, 30)
3	25	(3, 25)
4	20	(4, 20)
5	15	(5, 15)
6	10	(6, 10)
7	5	(7, 5)
8	0	(8, 0)

B.



10. A. 5 grams

B. Possible responses:
In symbols: $M = 40 - N \times 5$
In words: To find the mass of the food, multiply the number of days by five. Then subtract that number from 40 grams.

C. Possible responses: 40 grams of food is enough. Using the table, I see that the food will not be gone until Day 8.

Or, using my rule when N = 7:
$M = 40 - 7 \times 5$, so on Day 7,
M = 5 grams.

Teacher Guide

End-of-Year Test

Questions 1–21 (TG pp. 1–8)

1. 828
2. 5120
3. 1208
4. 364
5. 23,968
6. 4850
7. 29 R1
8. 147
9. **A.** Possible strategy: $100 \times 50 = 5000$

   B. Possible strategy: Count up 8 to 6,000. $6{,}000 \times 4 = 24{,}000$; $4 \times 8 = 32$; $24{,}000 - 32 = 23{,}968$.
10. **A.** About 1,300 pounds. $3{,}900 \div 3 = 1{,}300$.

    B. They saved about 34 trees. Possible solution: I rounded 3,908 to 4,000. 4,000 is twice as much as 2000. Since you save about 17 trees for every 2000 pounds of paper you will save about 34 trees with 4000 pounds.
11. **A.** 238, 396, 360, and 5050 are divisible by 2. They are all even numbers.

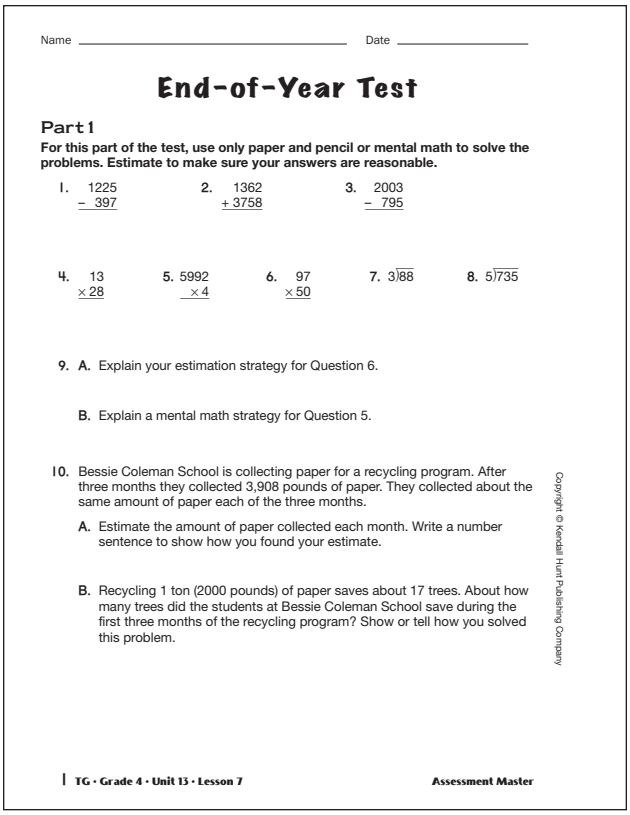
    B. 396, 360, 8235, 3063, and 4977 are divisible by 3. When you add the digits in each number they add up to a number divisible by 3 ($3 + 9 + 6 = 18$, 18 is divisible by 3 so 398 is divisible by 3.)

    C. 415, 360, 8235,and 5050 are divisible by 5. All of these numbers end in 5 or 0.

    D. 396, 360, are divisible by 6. Any number divisible by both 2 and 3 are divisible by 6.

    E. 396, 360, 8235, and 4977 are divisible by 9. When you add the digits in each number they add up to a number divisible by 9 ($8 + 2 + 3 + 5 = 18$, 18 is divisible by 9 so 8235 is divisible by 9).

    F. 360, 5050 are divisible by 10. All of the numbers end in 0.
12. Estimates will vary. 90°, 45°, 120°; accept answers within 10° larger or smaller.



Name _____ Date _____

End-of-Year Test

Part 1

For this part of the test, use only paper and pencil or mental math to solve the problems. Estimate to make sure your answers are reasonable.

1. 1225 − 397
2. 1362 + 3758
3. 2003 − 795
4. 13 × 28
5. 5992 × 4
6. 97 × 50
7. $3\overline{)88}$
8. $5\overline{)735}$

9. **A.** Explain your estimation strategy for Question 6.

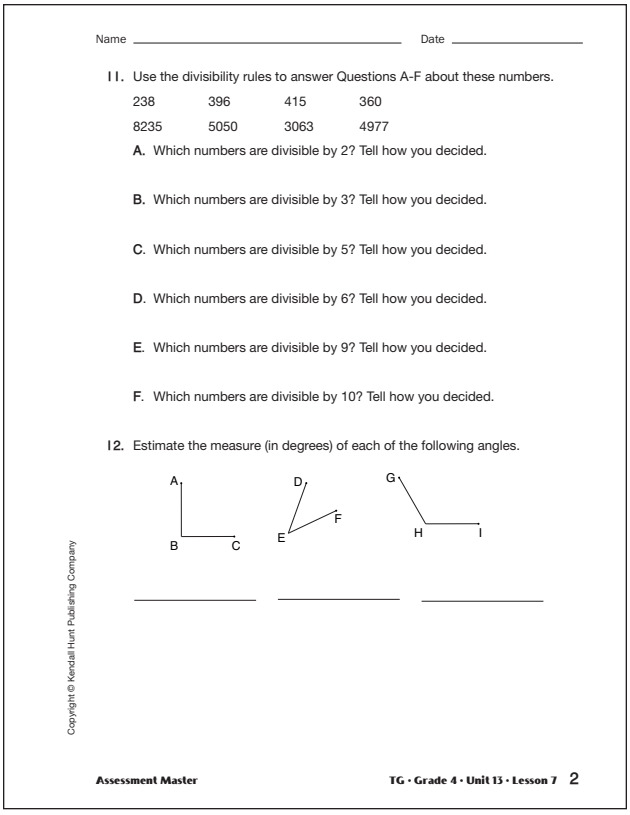
   B. Explain a mental math strategy for Question 5.

10. Bessie Coleman School is collecting paper for a recycling program. After three months they collected 3,908 pounds of paper. They collected about the same amount of paper each of the three months.

    A. Estimate the amount of paper collected each month. Write a number sentence to show how you found your estimate.

    B. Recycling 1 ton (2000 pounds) of paper saves about 17 trees. About how many trees did the students at Bessie Coleman School save during the first three months of the recycling program? Show or tell how you solved this problem.

***Teacher Guide* - Page 1**



Name _____ Date _____

11. Use the divisibility rules to answer Questions A-F about these numbers.

    238 396 415 360

    8235 5050 3063 4977

    A. Which numbers are divisible by 2? Tell how you decided.

    B. Which numbers are divisible by 3? Tell how you decided.

    C. Which numbers are divisible by 5? Tell how you decided.

    D. Which numbers are divisible by 6? Tell how you decided.

    E. Which numbers are divisible by 9? Tell how you decided.

    F. Which numbers are divisible by 10? Tell how you decided.

12. Estimate the measure (in degrees) of each of the following angles.

***Teacher Guide* - Page 2**

Name _____ Date _____

Part 2

Solve the following problems. You may use any of the tools that you usually use in class including a calculator.

13. Complete the following table. A flat ▱ is equal to 1.

Base-Ten Shorthand	Decimal	Fraction
▱▱ || ·····		$2\frac{25}{100}$
	1.6	
		$3\frac{4}{100}$
▱ ▱ ||| :·····		

14. Construct a quadrilateral ABCD using the following rules:

A. $\overleftrightarrow{AB}$ must be parallel to $\overleftrightarrow{DC}$

B. $\overleftrightarrow{AD}$ must be perpendicular to $\overleftrightarrow{DC}$

C. The measure of Angle A equals 90 degrees.

D. The measure of Angle B is less than 90 degrees.

15. Put the following fractions in order from smallest to largest.

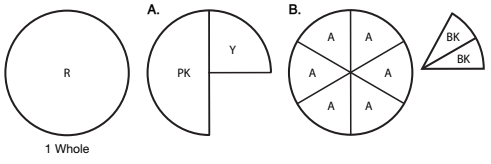
A. $\frac{1}{6}, \frac{1}{4}, \frac{1}{2}, \frac{1}{3}$

B. $\frac{4}{12}, \frac{1}{12}, \frac{13}{12}, \frac{14}{12}$

***Teacher Guide* - Page 3**

Name _____ Date _____

16. If a red circle is equal to 1 whole, name the following numbers each figure represents. You may use fraction circle pieces to help you.



_____ _____

17. Jackie ordered a special gift box of Chocos. There were 12 candies in the box.

A. $\frac{1}{3}$ of the candies in the box have cream filling.

B. $\frac{1}{6}$ of the candies in the box have caramel filling.

C. $\frac{3}{12}$ of the candies in the box are nutty clusters.

D. $\frac{1}{4}$ of the candies in the box have coconut filling.

How many of each kind are in Jackies's box? Complete the table. Write a multiplication number sentence.

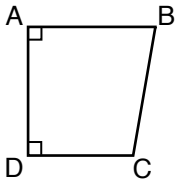
Kind of Candy	Number Sentence	Number of Candies in the Box
Cream Filling		
Caramel Filling		
Nutty Clusters		
Coconut Filling		

***Teacher Guide* - Page 4**

13.

Base-Ten Shorthand	Decimal	Fraction
▱▱ || ·····	2.25	$2\frac{25}{100}$
▱ ||||||	1.6	$1\frac{6}{10}$
▱▱▱ ····	3.04	$3\frac{4}{100}$
▱▱ ||| :·····	11.36	$11\frac{36}{100}$

14. One possible solution:



15. **A.** $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$

B. $\frac{1}{12}, \frac{4}{12}, \frac{13}{12}, \frac{14}{12}$

16. **A.** $\frac{3}{4}$

B. $\frac{7}{6}$ or $1\frac{1}{6}$ or $1\frac{2}{12}$

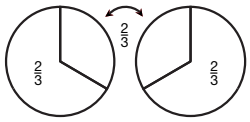
17.

Kind of Candy	Number Sentence	Number of Candies in the Box
Cream Filling	$\frac{1}{3}$ of 12 = 4 or $\frac{1}{3} \times 12 = \frac{12}{3} = 4$	4
Caramel Filling	$\frac{1}{6}$ of 12 = 2 or $\frac{1}{6} \times 12 = \frac{12}{6} = 2$	2
Nutty Clusters	$\frac{3}{12}$ of 12 = 3 or $\frac{3}{12} \times 12 = \frac{36}{12} = 3$	3
Coconut Filling	$\frac{1}{4}$ of 12 = 3 or $\frac{1}{4} \times 12 = \frac{12}{4} = 3$	3

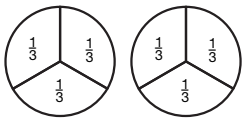
18.

	True	False
A. $3 \times \frac{2}{3} = 3 \times \frac{1}{2} \times 2$		✓
B. $6 \times \frac{1}{3} = 2$	✓	
C. $4 \times \frac{3}{8} = 2 \times 3$		✓
D. $4 \times \frac{3}{8} = \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$	✓	
E. $4 \times \frac{3}{8} = \frac{12}{8}$	✓	
F. $4 \times \frac{3}{8} = \frac{12}{32}$		✓
G. $3 \times \frac{2}{3} = 6 \times \frac{1}{3}$	✓	

H. Possible response: $3 \times \frac{2}{3} = \frac{6}{3} = 2$ and $6 \times \frac{1}{3} = \frac{6}{3} = 2$.



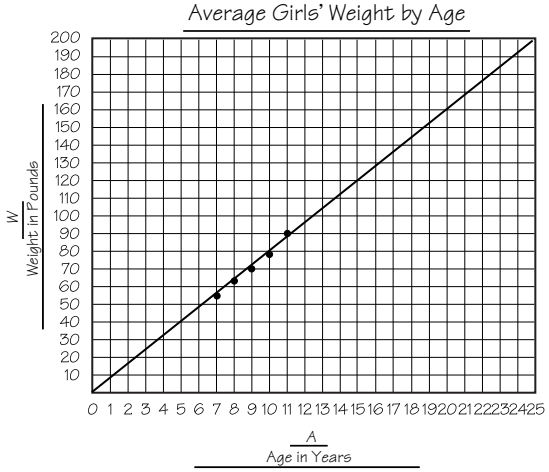
$3 \times \frac{2}{3} = \frac{6}{3}$



$6 \times \frac{1}{3} = \frac{6}{3}$

19. A. Possible response: The points slant up from left to right.

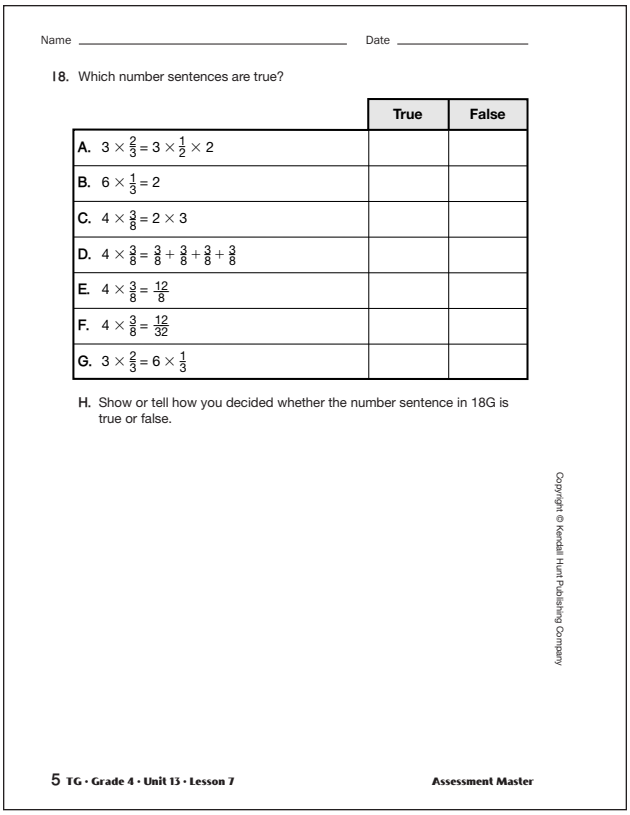
B.



C. Between 100 and 120 pounds (Estimates will vary.)

D. According to the graph, an average 23-year-old weighs between 180–190 pounds. Although a 23-year-old could weigh 180–190 pounds, this value is high for the average weight of a 23-year-old. Students should see that extrapolating this far beyond the last data point is unreliable.

E. According to the graph, an average 1-year-old weighs about 10 pounds. Students should see that extrapolating this far beyond the first data point is unreliable.



Name _____ Date _____

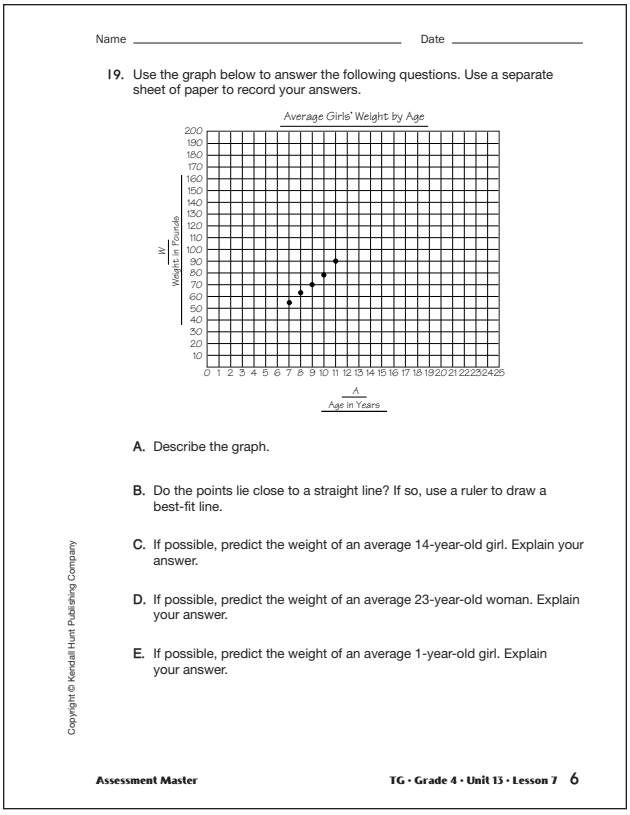
18. Which number sentences are true?

	True	False
A. $3 \times \frac{2}{3} = 3 \times \frac{1}{2} \times 2$		
B. $6 \times \frac{1}{3} = 2$		
C. $4 \times \frac{3}{8} = 2 \times 3$		
D. $4 \times \frac{3}{8} = \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$		
E. $4 \times \frac{3}{8} = \frac{12}{8}$		
F. $4 \times \frac{3}{8} = \frac{12}{32}$		
G. $3 \times \frac{2}{3} = 6 \times \frac{1}{3}$		

H. Show or tell how you decided whether the number sentence in 18G is true or false.

5 TG • Grade 4 • Unit 13 • Lesson 7 Assessment Master

***Teacher Guide* - Page 5**



Name _____ Date _____

19. Use the graph below to answer the following questions. Use a separate sheet of paper to record your answers.

A. Describe the graph.

B. Do the points lie close to a straight line? If so, use a ruler to draw a best-fit line.

C. If possible, predict the weight of an average 14-year-old girl. Explain your answer.

D. If possible, predict the weight of an average 23-year-old woman. Explain your answer.

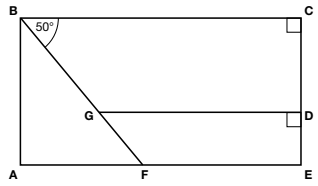
E. If possible, predict the weight of an average 1-year-old girl. Explain your answer.

Assessment Master TG • Grade 4 • Unit 13 • Lesson 7 6

***Teacher Guide* - Page 6**

Name _____ Date _____

20. Use the diagram below to answer Questions A–F below.



Find the measures of the following angles.

A. ∠ABF
B. ∠AFB
C. ∠CDG
D. ∠BGD
E. ∠DGF
F. ∠EFG
G. Show or tell how you found the answer for 20D.

***Teacher Guide* - Page 7**

Name _____ Date _____

21. Use the fraction circle pieces and the *Fraction Chart* to complete each table.

Multiply by $\frac{1}{3}$

A.

Input	Output
6	
9	
12	
15	

Multiply by $\frac{2}{3}$

B.

Input	Output
6	
9	
12	
15	

Multiply by $\frac{1}{5}$

C.

Input	Output
5	
10	
15	
20	

Multiply by $\frac{2}{5}$

D.

Input	Output
5	
10	
15	
20	

Multiply by $\frac{1}{4}$

E.

Input	Output
2	
4	
6	
8	
12	
16	

Multiply by $\frac{3}{4}$

F.

Input	Output
2	
4	
6	
8	
12	
16	

***Teacher Guide* - Page 8**

20. A. ∠ABF = 40°

B. ∠AFB = 50°

C. ∠CDG = 90°

D. ∠BGD = 130°

E. ∠DGF = 50°

F. ∠EFG = 130°

G. Possible response: if ∠CBG = 50° and ∠BCD = 90° that equals 140°; ∠CDG = 90° because $\overline{GD}$ is perpendicular to $\overline{CE}$; 140° + 90° = 230°, I know all 4 angles should sum to 360° so 360° – 230° = 130°.

21. A.

Input	Output
6	$\frac{6}{3} = 2$
9	$\frac{9}{3} = 3$
12	$\frac{12}{3} = 4$
15	$\frac{15}{3} = 5$

B.

Input	Output
6	$\frac{12}{3} = 4$
9	$\frac{18}{3} = 6$
12	$\frac{24}{3} = 8$
15	$\frac{30}{3} = 10$

C.

Input	Output
5	$\frac{5}{5} = 1$
10	$\frac{10}{5} = 2$
15	$\frac{15}{5} = 3$
20	$\frac{20}{5} = 4$

D.

Input	Output
5	$\frac{15}{5} = 3$
10	$\frac{30}{5} = 6$
15	$\frac{45}{5} = 9$
20	$\frac{60}{5} = 12$

E.

Input	Output
2	$\frac{2}{4} = \frac{1}{2}$
4	$\frac{4}{4} = 1$
6	$\frac{6}{4} = 1\frac{2}{4}$
8	$\frac{8}{4} = 2$
12	$\frac{12}{4} = 3$
16	$\frac{16}{4} = 4$

F.

Input	Output
2	$\frac{6}{4} = 1\frac{2}{4}$
4	$\frac{12}{4} = 3$
6	$\frac{18}{4} = 4\frac{2}{4}$
8	$\frac{24}{4} = 6$
12	$\frac{36}{4} = 9$
16	$\frac{48}{4} = 12$