Unit 1

MATHEMATICS IN THIS UNIT
Populations and Samples

Taken from the *Math Trailblazers* digital Teacher Guide

Students use bar graphs, line plots, and data tables to model the relationship between categorical and numerical variables. They use these tools as well as measures of central tendency (median and mode) to make generalizations and predictions about the populations they are studying.

Using the context of getting to know their classmates, the students design survey questions to collect information. Within this context they also review the use of variables and averages (median and mode) as they solve problems and review methods of collecting and representing data.

TIMS Laboratory Method. This unit reviews the four steps of the TIMS Laboratory Method: drawing a picture, collecting the data, graphing the data, and analyzing the results.

The four steps of the TIMS Laboratory Method will be familiar to students who have used *Math Trailblazers* before. Others will become accustomed to the routines as they gain experience with the method. Although we want students eventually to apply this method on their own, many may need to be guided at first. One of your more difficult instructional decisions will be how much guidance to give—how to balance imitation and autonomy. Learning by imitation will make for more orderly lessons, not an unimportant consideration in the beginning of the year. On the other hand, too much imitation can undermine student autonomy and can foster misconceptions about what mathematics and science are. As the year progresses, you will want to see students doing more work on their own, figuring out their strategies and solving problems with little teacher guidance.

Represent Data. In this unit, students will make and interpret bar graphs and line plots. The first lab, *Eyelets Lab* (Lesson1), will accommodate a range of approaches from teacher-directed to open-ended. In the main part of the lab you can be as directive as you think appropriate, especially since there is one data set for the whole class rather than a separate data set for each group.

The second lab, *Searching the Forest* (Lesson 4), demands more student autonomy since student groups gather their own data. Imitation is still important, but now every group's data is different. This means that only methods, not results, can be imitated.

One aspect of the method to stress in these early experiments is how the real objects—the picture, the data table(s), and the graph(s)—all represent the same situation. Many students benefit from connecting and comparing these various representations.

A great advantage of the TIMS Laboratory Method, with its multiple representations of mathematical ideas, is that it allows each student to work at his or her own level. Problems set in the rich context of a TIMS lab can often be solved by several methods, through mathematical reasoning, analysis of the graph or data tables, or even direct application of the laboratory apparatus. This allows a wide range of students to be accommodated by a single lesson. Discussing their solution methods also helps students develop their communication skills at the same time they are learning from one another.

Variables and Values. The term **variable** as used in this unit refers to attributes or quantities that change or vary in an experiment. Variables are basic to mathematics and science. Students should conceive the experiments as investigations about relationships between variables. *Eyelets Lab* can be seen as a study of how many pairs of shoes (*P*) there are with certain numbers of eyelets (*E*); *Searching the Forest* is a study of the number of tiles (*N*) of each color (*C*).

Several basic procedures for handling variables are involved in this unit: distinguishing between variables and values, denoting variables by symbols, and labeling data table columns and graphs axes with the names of variables. Students learn to use more specific terminology for variables—numerical and categorical variables. Variables that have numbers as their values are **numerical**; variables that have non-numerical values are **categorical**. For example, number of eyelets and number of pets are numerical variables because possible values for these variables (16 eyelets or 8 eyelets and 2 pets or 3 pets) are quantities. Categorical variables studied in this unit include favorite pet and tile color because possible values for these variables (dog or cat and red or blue) have to do with qualities, not quantities.

Averages. An **average** is a single value that represents a set of numbers. Although the mean is the most commonly used average in everyday activities, students learn to use three kinds of averages (mean, median, and mode) as part of their collection and analysis of data. Use of the mean is reviewed in a later unit. In this unit, the mode is introduced in Lessons 1 and 2 and the median is reviewed in Lessons 2 and 3. For more information on averages and how they are used, see the Content Note in Lesson 2.

Populations and Samples. The study of populations is the context for much of the work of this unit in which you will address the TIMS Laboratory Method, variables, and classroom climate. A **population** is a group of persons or things used as a base in statistical measurement. We can talk about the population of Chicago or Paducah, but we can also talk about the population of leaves on the General Sherman giant sequoia tree or the population of deer in a state.

Sometimes an entire population is small enough and well-defined enough for each individual to be counted or measured: the population of giant pandas in U.S. zoos, for example, can be counted or measured directly. More often, however, it is impossible, impractical, or unnecessary to count or measure an entire population: the population of giant pandas in the wilds of western China, for example, cannot be counted exactly. However, even when an entire population cannot be studied directly, there is often a way to gather useful information about it throught sampling.

A **sample** is a part or subset of a population. If you pull five leaves from a tree, you have a sample from the population of leaves on that tree; if you interview ten people leaving a polling place, then you have sampled the voting population at that location; if you pull a handful from a bowl of jellybeans, then you have sampled the jellybean population of that bowl. By studying a sample, one can often draw probable conclusions about the underlying population: If three of five oak leaves show evidence of a blight, then it's likely the tree has a problem; if eight of ten voters state that they voted for the incumbent, then that candidate is probably in good shape; if half the handful of jellybeans is red, then it's a good bet that about half of the beans in the bowl are red, as well.

Sampling is worth studying for its applications— public opinion polling, statistical process control, wildlife population estimation; indeed, much scientific research in general. It is also worth studying the interesting mathematics it requires: variables, classification, probability, mathematical reasoning, and statistics.

Math Trailblazers students begin studying populations and sampling in first grade when they pull handfuls of colored objects from grab bags. Population studies in the form of grab bags, frequency distributions, and classification labs are included in every later grade. The two labs in this unit are part of this series. The breadth and power of the mathematics of such population studies make a rich beginning for a productive school year.

Algebra in the Early Grades

There is a debate among mathematicians and educators about the content of algebra and especially the content of algebra in the early grades. Some have defined algebra as modeling, as pattern finding, as the study of structure, and as making sense of one's world quantitatively.

"We advocate for an early emphasis on developing children's ability to conceive of, reason about, and manipulate complex ideas and relationships, as an equal complement to numerical reasoning and computation. Children who develop a rich capacity for reasoning about general relationships among quantities will possess the conceptual foundation for learning and making sense of different programs and views of algebra." (Smith and Thompson, 2008)

This unit draws heavily on the connection between patterns and data. Through the use of tables, graphs, variables, and the TIMS Laboratory Method, students not only find ways to organize numerical and categorical data, they also discover patterns and relationships in those data. This focus on patterns captures the most central idea of early algebraic reasoning. The activities in this unit demonstrate the connections between the data strand and the algebra strand in *Math Trailblazers*.

This focus on early algebraic thinking is developed throughout *Math Trailblazers*. Following recommendations of researchers, activities are integrated through all grades that give students concepts and skills needed to make a natural transition from learning and doing arithmetic to learning and doing algebra. (Carpenter et al, 2003; Carraher and Schliemann, 2007; Kaput, 2008; Kilpatrick and Izsak, 2008; Schliemann et al, 2007). Students extend and connect their concepts of number, geometry, measurement, and data to develop tools for algebraic reasoning. These tools enable them to "do algebra," that is, to identify, describe, visualize, and simplify patterns and relationships. They learn to generalize procedures while they use arithmetic. They also learn to make generalizations about numbers that are collected in data sets, organized in tables, and pictured in graphs.

"Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved, attending to the meaning of the quantities . . ."

(Common Core State Standards Initiative, 2010)

Review and Practice

Every unit includes opportunities for distributed practice of concepts and skills. These resources can be found primarily in two places:

- Daily Practice and Problems (DPP) in the *Teacher Guide*
- Home Practice (HP) in the *Teacher Guide*

Daily Practice and Problems is a set of short exercises that provides ongoing review and study of math concepts and skills and provides a structure for systematically reviewing basic math facts. See the Daily Practice and Problems for this unit for more information about how they are organized and for ways to incorporate DPP items into your daily routine.

The Home Practice section of the *Teacher Guide* is a series of problems that supplements the homework included in the lessons. The Home Practice distributes skills practice throughout the units and reviews concepts studied in previous units.

Resources

- Carpenter, T.P., M.L. Franke, and L. Levi. *Thinking Mathematically.* Heinemann, Portsmouth, NH, 2003.
- Carraher, D.W. and A.D. Schliemann. "Early Algebra." In *Second Handbook of Research on Mathematics Teaching and Learning.* F.K. Lester, Jr., (Ed.). Information Age Publishing, Inc., Charlotte, NC, 2007.
- *Common Core State Standards for Mathematics.* National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington, DC, 2010.
- Cramer, K., M. Behr, T. Post, and R. Lesh. *Rational Number Project: Fraction Lessons for the Middle Grades - Level 1* <http://cehd.umn.edu/rationalnumberproject/rnp1.html>. Kendall Hunt Publishing Co., Dubuque, Iowa, 1997.
- Cramer, K., and T.R. Post. "Making Connections: A Case for Proportionality." In *Arithmetic Teacher.* 40(6), National Council of Teachers of Mathematics, Reston, VA, 1993.
- Cramer, K., T.R. Post, and R.C. delMas, "Initial Fraction Learning by Fourth- and Fifth-Grade Students: A Comparison of the Effects of Using Commercial Curricula With the Effects of Using the Rational Number Project Curriculum." In *Journal for Research in Mathematics Education.* 33(2), 111–144, National Council of Teachers of Mathematics, Reston, VA, 2002.
- Cramer, K., and T. Wyberg. "Efficacy of Different Concrete Models for Teaching the Part-Whole Construct for Fractions." *Mathematical Thinking and Learning*, 11(4), 226–257, Routledge, Taylor & Francis Group, 2012.
- Curcio, F.R., and N.S. Bezuk. *Understanding Rational Numbers and Proportions.* National Council of Teachers of Mathematics, Reston, VA, 1994.
- Kaput, J.J. *Algebra in the Early Grades.* Lawrence Erlbaum Associates, New York, NY, 2008.
- Kilpatrick, J. and A. Izsak. "A History of Algebra in the School Curriculum." In *Algebra and Algebraic Thinking in School Mathematics.* C.E. Greens and R. Rubenstein, (Eds.). The National Council of Teachers of Mathematics, Reston, VA, 2008.
- Lamon, Susan J. "Fractions and Rational Numbers." In *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers.* Lawrence Erlbaum Associates, Mahwah, New Jersey, 2006.
- Mack, N.K. "Learning Fractions with Understanding: Building on Informal Knowledge." In *Journal for Research in Mathematics Education.* 21(1), National Council of Teachers of Mathematics. Reston, VA, 1990.
- National Research Council. "Developing Proficiency With Other Numbers." In *Adding It Up: Helping Children Learn Mathematics.* J. Kilpatrick, J. Swafford, and B. Findell, (Eds.). National Academy Press, Washington D.C., 2001.
- Post, T.R., et al. "Order and Equivalence of Rational Numbers: A Cognitive Analysis." In *Journal for Research in Mathematics Education.* 16(1), National Council of Teachers of Mathematics. Reston, VA, 1990.
- *Principles and Standards for School Mathematics.* The National Council of Teachers of Mathematics, Reston, VA, 2000.
- Schliemann, A.D., D.W. Carraher, and B.M. Brizuela. *Bringing Out the Algebraic Character of Arithmetic*. Lawrence Erlbaum Associates, Publishers, Mahwah, NJ, 2007.
- Smith, John P. and P. Thompson. "Quantitative Reasoning and the Development of Algebraic Reasoning." In *Algebra in the Early Grades,* J. Kaput, D. Carraher, M. Blanton, (Eds.). Lawrence Erlbaum Associates, New York, NY, 2008.