Unit 3

MATHEMATICS IN THIS UNIT
Big Numbers

Taken from the *Math Trailblazers* digital Teacher Guide

Place Value and Number Sense. The major focus of this unit is on developing number sense for big numbers. A student's understanding of number sense occurs gradually. Students need opportunities to work with numbers in real-world contexts in order to develop an intuition about numbers. They also need an understanding of place value to be able to compute accurately and efficiently with large numbers. It is necessary to develop these basic number concepts as a foundation to understanding for computational procedures.

Students begin by reviewing place value concepts to read and write numbers into the billions. This leads to an exploration of how large numbers can be represented in multiple ways by flexible partitioning. These partitions are developed using place value charts, number lines, and written equations. Students then perform operations involving large numbers, including multiplication by multiples of ten and raising numbers to powers.

Estimation and Number Sense. Big numbers occur everyday in real life. The newspaper is filled with reports that are often in the billions. The population of the United States is in the hundreds of millions while that of China is well over a billion. The beginning activities in this unit focus on the magnitude of numbers in the millions and billions, so that students' work with large numbers is meaningful. As students develop a sense of perspective of the world around them, they begin to consider questions like:

- How many more people live in China than in the United States?
- How many times larger is the population of China than that of the United States?

In this unit, students compare and estimate large numbers by identifying the closest thousand, ten thousand, and so on. Problems that require ordering and estimating of large numbers develop a quantitative sense of how big these numbers are and how they compare to each other.

Estimation is a useful tool in our everyday lives that leads to better number sense. Students explore strategies for computational estimation in this unit. Students who are able to estimate well tend to be more flexible in their thinking, use a variety of estimation strategies, and demonstrate a deeper understanding of numbers and operations. Students use estimation as a means to find a "ballpark" answer when an exact answer is not necessary, when an exact answer is impossible to find, or to check whether calculated answers are reasonable. It is emphasized that estimation is particularly helpful when working with large numbers in real-life situations.

Computational estimation involves replacing the original number or numbers in a problem with more "convenient" numbers and then computing (usually mentally) with these numbers. For example, one apple costs 37 cents. To estimate the cost of 4 apples we might round 37 cents to 40 cents and then multiply 4×40 cents mentally. The convenient number we chose here was 40 cents. The convenient number may be found by formal rounding procedures, but it does not always have to be. For instance, to multiply 76×4, it may be more convenient to estimate the product as 75×4, which is 75 doubled ($75 \times 2 = 150$) and doubled again ($150 \times 2 = 300$). This choice of a convenient number demonstrates a flexible, efficient method to finding an answer that is reasonably close.

Computational estimation is an important life skill. When discussing estimation, it is important to emphasize that different estimates may be reasonable.

Place Value and Multiplication

In a later unit, students will review and extend strategies for multidigit multiplication. The ability to multiply by multiples of ten plays an important role in developing these strategies and in estimating products involving large numbers.

Many students likely already know how to apply a rule for multiplying numbers ending in zero that involves multiplying the leading parts of the numbers and then appending the trailing zeros to the product. However, many students may not readily understand the mathematics behind this rule. They may describe "adding zeros to the end" without understanding the place value meaning of multiplying by multiples of ten. In this unit, students explore this concept using number lines, area models, and computational models that emphasize place value. Once we have established the mathematical meaning of multiplying multiples of ten, students then develop the rule for appending zeros and apply it to solve problems using mental math.

Partitioning Large Numbers

The ability to decompose and recompose numbers helps students develop computational strategies. The partitioning of numbers by place-value (e.g., 1234 = 1000 + 200 + 30 + 4) is especially important when computing with larger numbers. For example, finding the sum of 1234 + 6255 can be done efficiently by grouping and regrouping sums according to place value. At the same time, students should also be able to partition numbers with flexibility, so that they are not limited to partitioning by place value only.

Students explore partitioning using the number line. As they find different ways to "get to" a number on the number line, they are also finding multiple ways to partition that number. Figure 1 shows two different ways that 6098 can be partitioned using a number line. Each way can also be written as a mathematical expression. Students see how these equivalent expressions form number sentences, or equations.

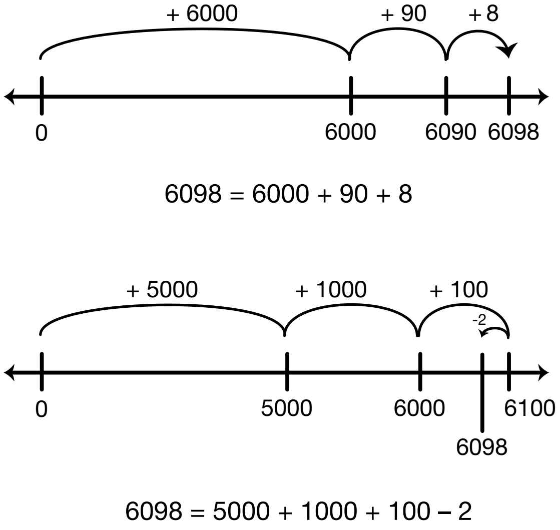

Figure 1: *Two ways to partition 6098*

When unknowns are introduced into partitions as shown in the number sentence below, students are able to explore algebraic concepts as well.

$1976 + 1000 + n + 70 + 6$

An understanding of how numbers can be partitioned allows students to think of an unknown first as a "missing part," rather than as a number to find via the formal rules of isolating variables.

Math Facts and Mental Math

This unit continues the review and assessment of the division facts to develop mental math strategies, gain fluency, and to learn to apply multiplication and division strategies to larger numbers. Students will focus on the division facts for the 2s and 3s.

Resources

- *Assessment Standards for School Mathematics.* National Council of Teachers of Mathematics, Reston, VA, 1989.
- Fuson, K. "Research on Whole Number Addition and Subtraction." In *Handbook of Research on Mathematics Teaching and Learning.* pp. 243–275, D.A. Grouws (Ed.). Macmillan Publishing Company, New York, 1992.
- Hiebert, J., T. Carpenter, E. Fennema, K. Fuson, D. Wearne, H. Murray, A. Olivier, and H. Piet. *Making Sense: Teaching and Learning Mathematics with Understanding.* Heinemann, Portsmouth, 1997.
- Isaacs, A. C., and W. M. Carroll. "Strategies for Basic-Facts Instruction." *Teaching Children Mathematics,* 5 (9), pp. 508–515, National Council of Teacher of Mathematics, Reston, VA, 1999.
- Kilpatrick, J., J. Swafford, and B. Findell (Eds.). *Adding It Up: Helping Children Learn Mathematics.* National Academy Press, Washington, DC, 2001.
- Payne, J.N. (Ed.). *Mathematics for the Young Child.* National Council of Teachers of Mathematics, Reston, VA, 1990.
- *Principles and Standards for School Mathematics,* National Council of Teachers of Mathematics, Reston, VA, 2000.
- Sowder, J. *Estimation and Number Sense.* In *Handbook of Research on Mathematics Teaching and Learning,* pp. 371–389, D.A. Grouws (Ed.). Macmillan Publishing Company, New York, 1992.
- Stenmark, J.K. (Ed.). *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions.* National Council of Teachers of Mathematics, Reston, VA, 1991.
- Stigler, J.W., S.Y. Lee, and H.W. Stevenson. *The Mathematical Knowledge of Japanese, Chinese, and American Elementary School Children.* National Council of Teachers of Mathematics, Reston, VA, 1990.