Equivalent Fractions and Ratios

Grace's Cookies

Whenever Grace's grandmother comes to visit, she brings cookies for Grace to share with her three brothers. Last week she brought four cookies, so Grace and her brothers got one cookie each.

"Today I brought 8 cookies for the four of you," said Grace's grandmother.

"How many is that for each of us?" asked Grace's youngest brother.

"Well, we have to share the cookies equally among the four of us," said Grace. "If there are eight cookies total, that means we each get two cookies, since $4 \times 2 = 8$."

"That's right," said Grandma. "And next week, I will bring 12 cookies. How many will you each get then?"

Discuss

1. How many cookies will Grace and her brothers each get if their grandmother brings 12 cookies? Explain how you know.

Grace's grandmother says that for each of the next three weeks she will bring four more cookies than the week before. Grace makes a table to show the number of cookies her grandmother brings, starting with the first week.

***Student Guide* - Page 216**

2. Draw the table below. Fill in the second column of the table to show how many cookies Grace's grandmother will bring on the 4th week and 5th week.

Grace's Share of Cookies

Week	Total Number of Cookies	Number of Cookies Grace Gets
1st week	4	1
2nd week	8	2
3rd week	12	
4th week		
5th week		

3. How many cookies will Grace get in the 4th week and 5th week if she shares the cookies equally with her three brothers? Explain how you know.
4. Fill in the third column of the table with your answers from Questions 1 and 3.
5. Make a point graph. Plot the total number of cookies on the horizontal axis and the number of cookies Grace gets on the vertical axis. Remember to title the graph and label the axes. Use a piece of *Centimeter Graph Paper.*
6. If the points form a line, draw a line through the points.
   - **A.** What does the graph tell you about the number of cookies Grandma brings each week?
   - **B.** How does the graph show it?
7. What does the graph tell you about the relationship between the number of cookies Grace gets each week and the total number of cookies?
8. Use the graph to figure out how many cookies Grace gets if her grandmother brings 28 cookies.

In the data table of Grace's share of cookies, data is shown **vertically.** That means the data is written in columns. Data tables can also be shown **horizontally,** where the data is shown in rows. Mathematicians sometimes use horizontal tables to show relationships in data. Grace wants to use a horizontal table to show how the numbers of cookies are related as a ratio.

***Student Guide* - Page 217**

Student Guide

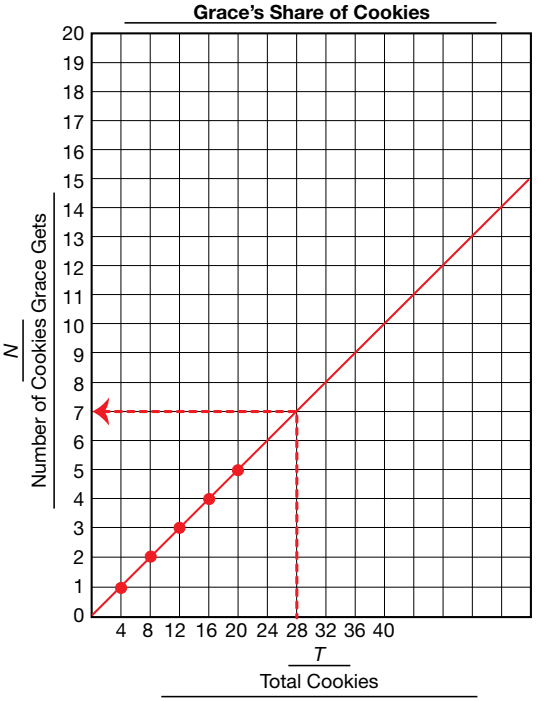
Equivalent Fractions and Ratios Questions 1–20 (SG pp. 216–220)

1. 3 cookies. Explanations may vary. $4 \times 3 = 12$.
2.

Grace's Share of Cookies

Week	Total Number of Cookies	Number of Cookies Grace Gets
1st week	4	1
2nd week	8	2
3rd week	12	3
4th week	16	4
5th week	20	5

3. In the 4th week Grace gets 4 cookies; in the 5th week, she gets 5 cookies. Explanations may vary. When Grandma makes 4 more cookies than she did the previous week, that means each child gets 1 more cookie than the previous week.
4. See table in Question 2.
5–6.



6. **A–B.** Grandma increases by 4 how many cookies she bakes each week. Explanations may vary.
7. As the total number of cookies increases by 4, the number of cookies Grace receives increases by 1.
8. 7 cookies

9.

Grace's Share of Cookies

Number of Cookies Grace Gets	1	2	3	4	5
Total Number of Cookies	4	8	12	16	20

10. Grace gets $\frac{1}{4}$ of the total cookies.

11. $\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}$

12. Explanations may vary. The denominator is 4 times greater than the numerator, or the numerator is $\frac{1}{4}$ the denominator.

13. Explanations may vary. Grace gets one more cookie each week that Grandma bakes 4 more cookies. The number increases.

14. The value of the fractions stays the same. $\frac{1}{4}, \frac{2}{8}, \frac{3}{12}$ etc. are equivalent fractions.

15.* No. Explanations may vary. Think of the benchmark $\frac{1}{2}$. $\frac{1}{4}$ is less than $\frac{1}{2}$. $\frac{6}{9}$ is greater than $\frac{1}{2}$.

16.* Yes. Explanations may vary. Josh needs to multiply 1×6 to get a numerator of six. Then he can multiply the denominator times 6. This way we will find the equivalent fraction.

17. Answers may vary. $\frac{8}{12}, \frac{12}{16}$

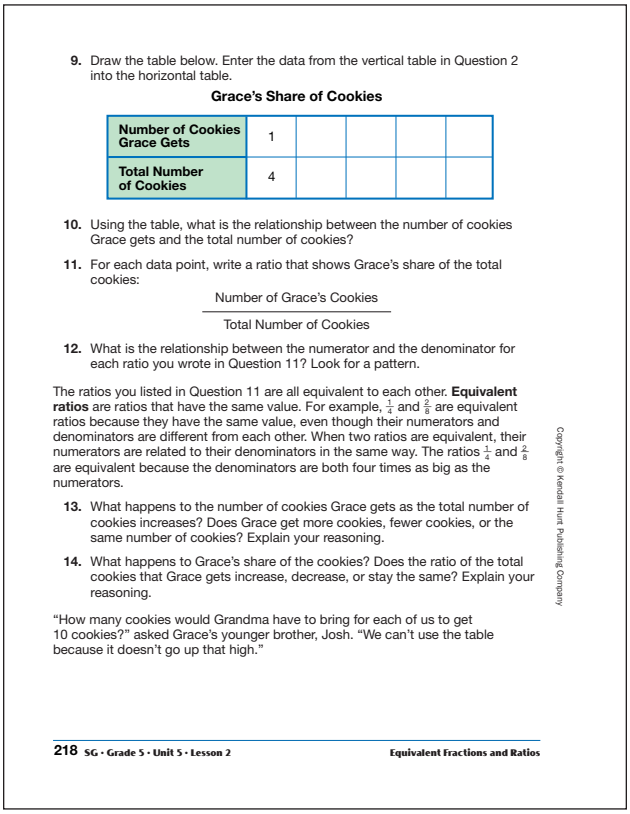
18. Answers may vary.

A. $\frac{4}{10}, \frac{6}{15}$

B. $\frac{6}{8}, \frac{9}{12}$

C. $\frac{4}{6}, \frac{2}{3}$

D. $\frac{6}{4}, \frac{9}{6}$



9. Draw the table below. Enter the data from the vertical table in Question 2 into the horizontal table.

Grace's Share of Cookies

Number of Cookies Grace Gets	1				
Total Number of Cookies	4				

10. Using the table, what is the relationship between the number of cookies Grace gets and the total number of cookies?

11. For each data point, write a ratio that shows Grace's share of the total cookies:

$$\frac{\text{Number of Grace's Cookies}}{\text{Total Number of Cookies}}$$

12. What is the relationship between the numerator and the denominator for each ratio you wrote in Question 11? Look for a pattern.

The ratios you listed in Question 11 are all equivalent to each other. **Equivalent ratios** are ratios that have the same value. For example, $\frac{1}{4}$ and $\frac{2}{8}$ are equivalent ratios because they have the same value, even though their numerators and denominators are different from each other. When two ratios are equivalent, their numerators are related to their denominators in the same way. The ratios $\frac{1}{4}$ and $\frac{2}{8}$ are equivalent because the denominators are both four times as big as the numerators.

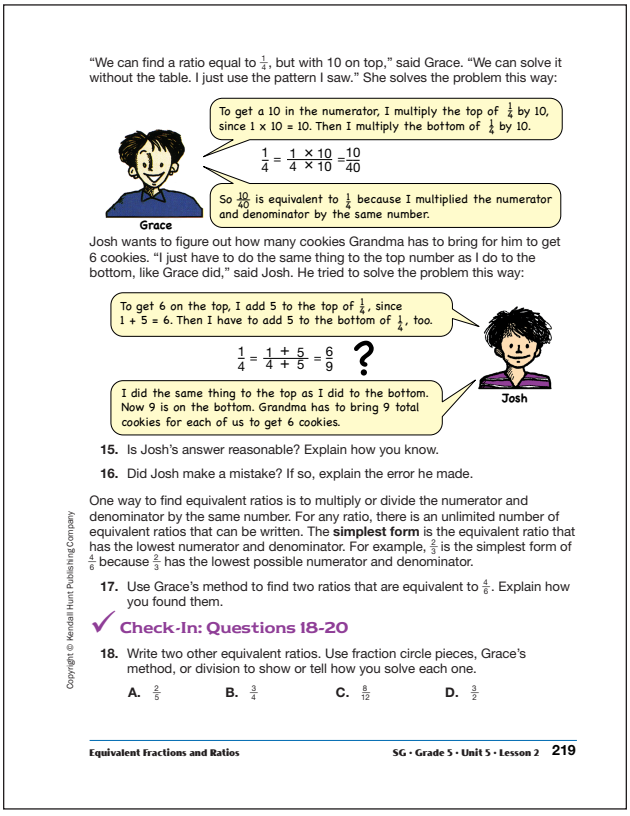
13. What happens to the number of cookies Grace gets as the total number of cookies increases? Does Grace get more cookies, fewer cookies, or the same number of cookies? Explain your reasoning.

14. What happens to Grace's share of the cookies? Does the ratio of the total cookies that Grace gets increase, decrease, or stay the same? Explain your reasoning.

"How many cookies would Grandma have to bring for each of us to get 10 cookies?" asked Grace's younger brother, Josh. "We can't use the table because it doesn't go up that high."

218 SG • Grade 5 • Unit 5 • Lesson 2 Equivalent Fractions and Ratios

***Student Guide* - Page 218**



"We can find a ratio equal to $\frac{1}{4}$, but with 10 on top," said Grace. "We can solve it without the table. I just use the pattern I saw." She solves the problem this way:

Grace

To get a 10 in the numerator, I multiply the top of $\frac{1}{4}$ by 10, since 1 x 10 = 10. Then I multiply the bottom of $\frac{1}{4}$ by 10.

$\frac{1}{4} = \frac{1 \times 10}{4 \times 10} = \frac{10}{40}$

So $\frac{10}{40}$ is equivalent to $\frac{1}{4}$ because I multiplied the numerator and denominator by the same number.

Josh wants to figure out how many cookies Grandma has to bring for him to get 6 cookies. "I just have to do the same thing to the top number as I do to the bottom, like Grace did," said Josh. He tried to solve the problem this way:

To get 6 on the top, I add 5 to the top of $\frac{1}{4}$, since 1 + 5 = 6. Then I have to add 5 to the bottom of $\frac{1}{4}$, too.

$\frac{1}{4} = \frac{1+5}{4+5} = \frac{6}{9}$?

I did the same thing to the top as I did to the bottom. Now 9 is on the bottom. Grandma has to bring 9 total cookies for each of us to get 6 cookies.

Josh

15. Is Josh's answer reasonable? Explain how you know.

16. Did Josh make a mistake? If so, explain the error he made.

One way to find equivalent ratios is to multiply or divide the numerator and denominator by the same number. For any ratio, there is an unlimited number of equivalent ratios that can be written. The **simplest form** is the equivalent ratio that has the lowest numerator and denominator. For example, $\frac{2}{3}$ is the simplest form of $\frac{1}{6}$ because $\frac{2}{3}$ has the lowest possible numerator and denominator.

17. Use Grace's method to find two ratios that are equivalent to $\frac{4}{6}$. Explain how you found them.

✓ **Check-In: Questions 18-20**

18. Write two other equivalent ratios. Use fraction circle pieces, Grace's method, or division to show or tell how you solve each one.

A. $\frac{2}{5}$ **B.** $\frac{3}{4}$ **C.** $\frac{8}{12}$ **D.** $\frac{3}{2}$

Equivalent Fractions and Ratios SG • Grade 5 • Unit 5 • Lesson 2 219

***Student Guide* - Page 219**

***Answers and/or discussion are included in the lesson.**

19. For each of the fractions below, use the *Multiplication Facts I Know* chart to find two other equivalent fractions.

A. $\frac{2}{3}$ B. $\frac{8}{10}$ C. $\frac{1}{7}$ D. $\frac{4}{8}$

E. $\frac{9}{2}$ F. $\frac{7}{9}$ G. $\frac{3}{7}$ H. $\frac{8}{6}$

20. Explain why using the *Multiplication Facts I Know* chart works to find equivalent fractions.

Homework

You will need a piece of graph paper to complete the homework.

1. Jacob notices that each candy bar he eats has three almonds on it.

Almonds in the Bars

Number of Candy Bars	Total Number of Almonds
1	3
2	
3	
4	
5	

   A. Fill in the table that shows the number of almonds compared to the number of candy bars.
   B. Make a point graph with the number of candy bars on the horizontal axis and the total number of almonds on the vertical axis. Be sure to title your graph and label your axes.
   C. If the points lie on a line, draw a line through the points.
   D. Use the line to find the total number of almonds on 8 candy bars.
   E. For each point on your graph, write a fraction that shows the number of almonds (numerator) compared to the number of candy bars (denominator).
   F. Which fraction from Question E is in simplest form?
   G. Write three more equivalent fractions that compare the number of almonds to the number of candy bars.

2. Write each fraction below on your paper. Find two equivalent fractions for each. Show or tell how you found the equivalent fractions. Circle each fraction you think is in simplest form or write the fraction in simplest form.

A. $\frac{10}{5}$ B. $\frac{1}{3}$ C. $\frac{3}{8}$ D. $\frac{9}{6}$

E. $\frac{5}{2}$ F. $\frac{2}{6}$ G. $\frac{4}{12}$ H. $\frac{2}{5}$

***Student Guide* - Page 220**

19. Answers may vary.

A.* $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}$

B. $\frac{16}{20}, \frac{24}{30}, \frac{32}{40}$

C. $\frac{2}{14}, \frac{3}{21}, \frac{4}{28}$

D. $\frac{8}{16}, \frac{12}{24}, \frac{16}{32}$

E. $\frac{18}{4}, \frac{27}{6}, \frac{32}{8}$

F. $\frac{14}{18}, \frac{21}{27}, \frac{28}{32}$

G. $\frac{6}{14}, \frac{12}{28}, \frac{18}{42}$

H. $\frac{16}{12}, \frac{24}{18}, \frac{32}{24}$

20.* Explanations may vary. If you look at any row or column of the *Multiplication Facts I Know* chart, you will see that a number increases by its multiples. Choose a fraction. Find its numerator and its denominator. If from the numerator you move 1 box to the right, do the same for the denominator, and you will have the numbers to create an equivalent fraction.

Homework (SG p. 12)

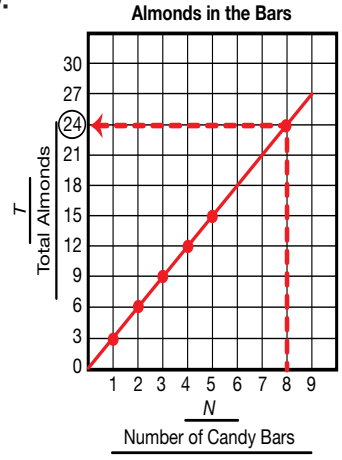
Questions 1–2

1. A.

Almonds in the Bars

Number of Candy Bars	Total Number of Almonds
1	3
2	6
3	9
4	12
5	15

B–C.



D. 24 almonds

E. $\frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \frac{12}{4}, \frac{15}{5}$

F. $\frac{3}{1}$

G. Answers may vary. $\frac{18}{6}, \frac{21}{7}, \frac{24}{8}, \frac{30}{10}$

2. Answers may vary.

A. $\frac{10}{5}, \left(\frac{2}{1}\right), \frac{20}{10}$

B. $\left(\frac{1}{3}\right), \frac{2}{6}, \frac{10}{30}$

C. $\left(\frac{3}{8}\right), \frac{6}{16}, \frac{9}{24}$

D. $\frac{9}{6}, \left(\frac{3}{2}\right), \frac{90}{60}$

E. $\left(\frac{5}{2}\right), \frac{10}{4}, \frac{15}{6}$

F. $\frac{2}{6}, \left(\frac{1}{3}\right), \frac{10}{9}, \frac{4}{12}$

G. $\frac{4}{12}, \left(\frac{1}{3}\right), \frac{100}{300}$

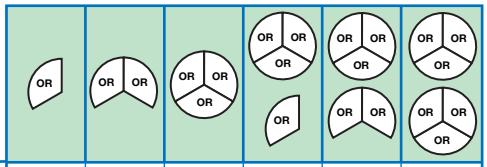
H. $\left(\frac{2}{5}\right), \frac{4}{10}, \frac{8}{20}$

*Answers and/or discussion are included in the lesson.

Student Activity Book

Fraction Cover-Up 1 (SAB pp. 197–198)
Questions 1–10

1.* TABLE 1

						
Number of Orange Pieces	1	2	3	4	5	6
Number of Aqua Pieces	2	4	6	8	10	12
Fraction: Number of Orange / Number of Aqua	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$	$\frac{6}{12}$

2.* The denominator is 2 times larger than the numerator.

3.* Yes. Explanations may vary. In each example, if you take $\frac{1}{2}$ and multiply both the numerator and the denominator by the same thing, you will get an equivalent fraction.

4.* $\frac{1}{2}$. The fewest number of fraction pieces makes it.

5. Possible responses: 1, 2; 2, 4; or 3, 6, etc.

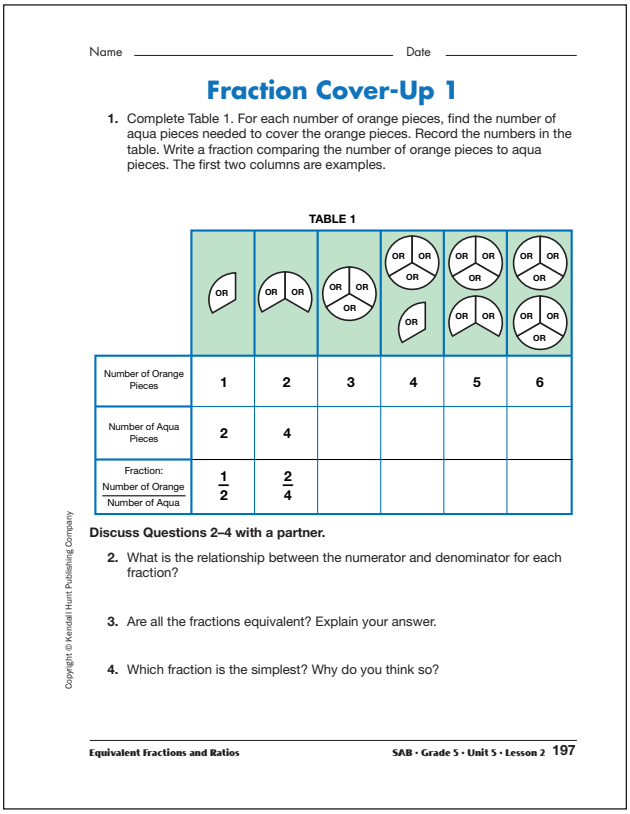
6. 1

7. Possible responses: $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, etc.

8. Possible responses: $\frac{2}{1}$, $\frac{4}{2}$, $\frac{6}{4}$, etc.

9. $\frac{2}{1}$

10. $\frac{1}{2}$



Name _____ Date _____

Fraction Cover-Up 1

1. Complete Table 1. For each number of orange pieces, find the number of aqua pieces needed to cover the orange pieces. Record the numbers in the table. Write a fraction comparing the number of orange pieces to aqua pieces. The first two columns are examples.

TABLE 1

Number of Orange Pieces	1	2	3	4	5	6
Number of Aqua Pieces	2	4				
Fraction: Number of Orange / Number of Aqua	$\frac{1}{2}$	$\frac{2}{4}$				

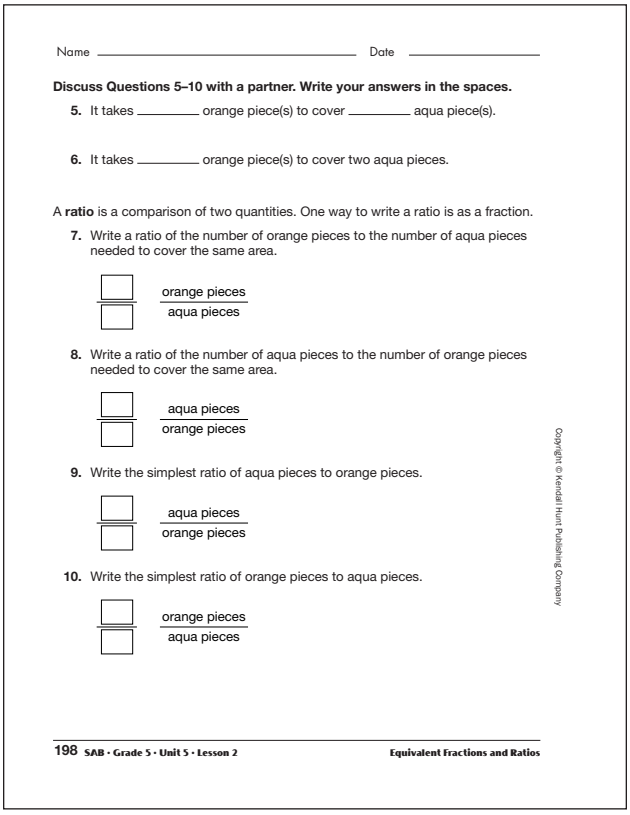
Discuss Questions 2–4 with a partner.

2. What is the relationship between the numerator and denominator for each fraction?

3. Are all the fractions equivalent? Explain your answer.

4. Which fraction is the simplest? Why do you think so?

***Student Activity Book* - Page 197**



Name _____ Date _____

Discuss Questions 5–10 with a partner. Write your answers in the spaces.

5. It takes _____ orange piece(s) to cover _____ aqua piece(s).

6. It takes _____ orange piece(s) to cover two aqua pieces.

A **ratio** is a comparison of two quantities. One way to write a ratio is as a fraction.

7. Write a ratio of the number of orange pieces to the number of aqua pieces needed to cover the same area.

$\frac{\square}{\square}$ orange pieces / aqua pieces

8. Write a ratio of the number of aqua pieces to the number of orange pieces needed to cover the same area.

$\frac{\square}{\square}$ aqua pieces / orange pieces

9. Write the simplest ratio of aqua pieces to orange pieces.

$\frac{\square}{\square}$ aqua pieces / orange pieces

10. Write the simplest ratio of orange pieces to aqua pieces.

$\frac{\square}{\square}$ orange pieces / aqua pieces

***Student Activity Book* - Page 198**

***Answers and/or discussion are included in the lesson.**

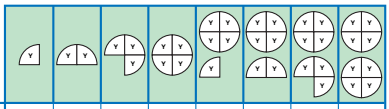
Name _____ Date _____

Fraction Cover-Up 2

1. Complete Table 2. For each row A–E:
   - Find the number of pieces of each color it takes to cover the shape in the top row exactly.
   - In each column, record the fraction of the number of yellow pieces to the number of pieces of the second color. Write an "x" if one of the shapes could not be covered exactly.

   Follow the examples.

TABLE 2



A. Number of Yellow / Number of Blue	$\frac{1}{2}$	$\frac{2}{4}$						
B. Number of Yellow / Number of Pink	×	$\frac{2}{1}$						
C. Number of Yellow / Number of Red								
D. Number of Yellow / Number of Black								
E. Number of Yellow / Number of Orange								

***Student Activity Book* - Page 199**

Name _____ Date _____

Discuss Questions 2–8 with a partner.

2. Which boxes could you not write a fraction for? Why not?

3. What patterns do you notice about the fractions in each row?

4. What is the relationship between the numerator and denominator for each fraction in Row B?

5. Are all the fractions in Row B equivalent? Explain your answer.

6. Are all the fractions in Row E equivalent? Explain your answer.

7. In which rows are the fractions greater than one?

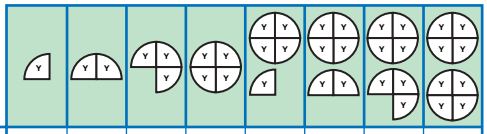
8. Which fraction is the simplest in each row? Why do you think so?

***Student Activity Book* - Page 200**

Fraction Cover-Up 2 (SAB pp. 199–201)
Questions 1–14

1. A–E.

TABLE 2



A. Number of Yellow / Number of Blue	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$	$\frac{6}{12}$	$\frac{7}{14}$	$\frac{8}{16}$
B. Number of Yellow / Number of Pink	×	$\frac{2}{1}$	×	$\frac{4}{2}$	×	$\frac{6}{3}$	×	$\frac{8}{4}$
C. Number of Yellow / Number of Red	×	×	×	$\frac{4}{1}$	×	×	×	$\frac{8}{2}$
D. Number of Yellow / Number of Black	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{3}{9}$	$\frac{4}{12}$	$\frac{5}{15}$	$\frac{6}{18}$	$\frac{7}{21}$	$\frac{8}{24}$
E. Number of Yellow / Number of Orange	×	×	×	$\frac{4}{3}$	×	×	×	$\frac{8}{6}$

2.* See chart. You can't cover the two colors evenly.

3. When moving from 1 equivalent fraction to the next the increase in the numerator is consistent and the increase in the denominator is consistent.

4. The numerator is 2 times greater than the denominator.

5. Yes. Explanations may vary. The numerator equals 2 times the denominator.

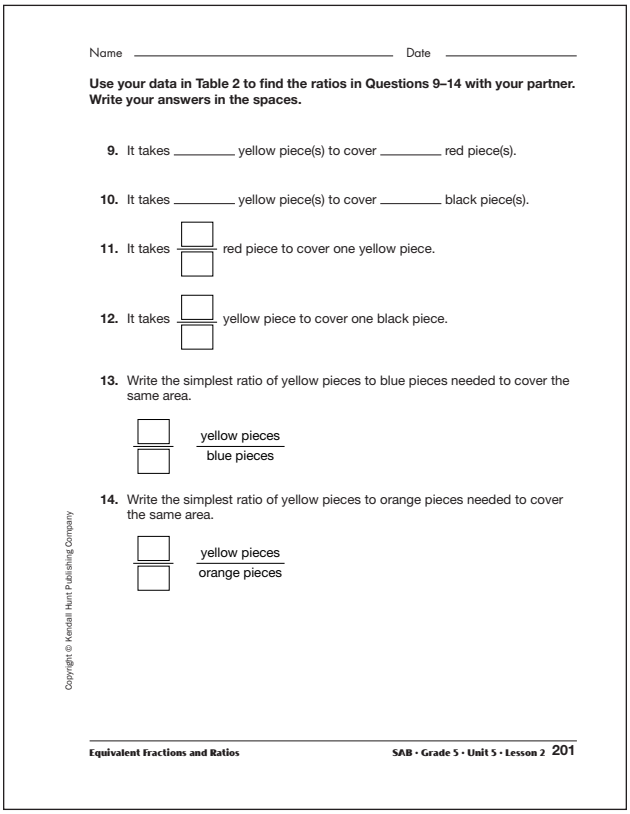
6. Yes. Explanations may vary. For every 4 yellow pieces, I need 3 orange pieces.

7. Rows B, C, and E

8. The first one listed. Explanations may vary. We look for the relationship using the fewest possible pieces.

***Answers and/or discussion are included in the lesson.**

9. 4 yellow; 1 red
10. 1 yellow; 3 blacks
11. $\frac{1}{4}$
12. $\frac{1}{3}$
13. $\frac{1}{2}$
14. $\frac{4}{3}$



Name _____ Date _____

Use your data in Table 2 to find the ratios in Questions 9–14 with your partner. Write your answers in the spaces.

9. It takes _____ yellow piece(s) to cover _____ red piece(s).

10. It takes _____ yellow piece(s) to cover _____ black piece(s).

11. It takes $\frac{\square}{\square}$ red piece to cover one yellow piece.

12. It takes $\frac{\square}{\square}$ yellow piece to cover one black piece.

13. Write the simplest ratio of yellow pieces to blue pieces needed to cover the same area.

$\frac{\square}{\square}$ yellow pieces / blue pieces

14. Write the simplest ratio of yellow pieces to orange pieces needed to cover the same area.

$\frac{\square}{\square}$ yellow pieces / orange pieces

***Student Activity Book* - Page 201**

Name _____ Date _____

Fraction Cover-Up 3

1. Complete Table 3. For each row A–E:
   - Find the number of pieces of each color it takes to cover the shape in the top row exactly.
   - In each column, record the fraction of the number of yellow pieces to the number of pieces of the second color. Write an "x" if one of the shapes could not be covered exactly.

   Follow the examples.

TABLE 3

						
A. Number of Purple / Number of Green	$\frac{2}{1}$	×				
B. Number of Purple / Number of Yellow	×	$\frac{5}{2}$				
C. Number of Purple / Number of Orange						
D. Number of Purple / Number of Pink						
E. Number of Purple / Number of Red						

Discuss Questions 2–3 with a partner.

2. Which boxes could you not write a fraction for? Why not?

***Student Activity Book* - Page 203**

Name _____ Date _____

3. Decimal fractions also represent fractional parts. Use the red circle as the unit whole.

   A. Work with your partner to show the following decimal fractions with a combination of purple, green, yellow, orange, or pink pieces.

   .5 1.2 .8

   B. Write a common fraction next to the decimal fraction.

   .5 = ☐/☐ 1.2 = ☐/☐ .8 = ☐/☐

Use your data in Table 3 and your fraction circle pieces to find the fractions and ratios in Questions 4–12. The red circle is the unit whole, and decimal fractions represent fractional parts.

4. It takes _____ purple pieces to cover one red piece.

5. **A.** One purple piece covers what fraction of the red circle? ☐/☐

   B. Write a decimal fraction for one purple piece. _____

***Student Activity Book* - Page 204**

Fraction Cover-Up 3 (SAB pp. 203–206)
Questions 1–12

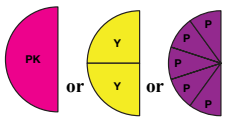
1. A–E.

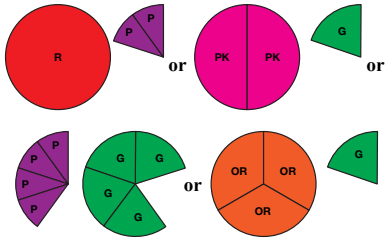
TABLE 3

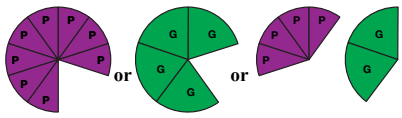
						
A. Number of Purple / Number of Green	$\frac{2}{1}$	×	$\frac{8}{4}$	$\frac{10}{5}$	×	$\frac{20}{10}$
B. Number of Purple / Number of Yellow	×	$\frac{5}{2}$	×	$\frac{10}{4}$	$\frac{15}{6}$	$\frac{20}{8}$
C. Number of Purple / Number of Orange	×	×	×	$\frac{10}{3}$	×	$\frac{20}{6}$
D. Number of Purple / Number of Pink	×	$\frac{5}{1}$	×	$\frac{10}{2}$	$\frac{15}{3}$	$\frac{20}{4}$
E. Number of Purple / Number of Red	×	×	×	$\frac{10}{1}$	×	$\frac{20}{2}$

2.* See chart. You can't cover the two colors evenly.

3. **A.*** Answers will vary.

.5 = 

1.2 = 

.8 = 

B. .5 = $\frac{5}{10}$, $\frac{1}{2}$, $\frac{2}{4}$, etc.
1.2 = $1\frac{2}{10}$, $1\frac{1}{5}$, $\frac{12}{10}$, or $\frac{6}{5}$ etc.
.8 = $\frac{8}{10}$, $\frac{4}{5}$

4. 10 purple pieces

5. **A.** $\frac{1}{10}$
   B. .1

***Answers and/or discussion are included in the lesson.**