Unit 8

MATHEMATICS IN THIS UNIT
Decimals

Taken from the *Math Trailblazers* digital Teacher Guide

Fraction quantities can be represented numerically using either common fractions or decimal fractions. Students are familiar with common fractions, expressed with a numerator and denominator as with $\frac{1}{2}$, $\frac{3}{5}$, $\frac{7}{10}$, $2\frac{5}{8}$, or $\frac{9}{4}$. Decimal fractions, or decimals, express fractional quantities by including digits to the right of a decimal point, as with 0.5, 0.046, or 2.33. Decimals make use of base-ten place-value concepts to symbolize fractional quantities, which makes them particularly useful for performing computations and recording measurements.

The Meaning of Decimals

Students make sense of decimal fractions by connecting decimal concepts to their existing understanding of common fractions and by exploring various representations of decimal fractions. For example, students learn the meaning of decimals by reading "4.3" aloud as "four and three-tenths," writing 1.36 as $1 + \frac{3}{10} + \frac{6}{100}$, or seeing that $\frac{1}{2}$ of a meter and 0.5 meter occupy the same location on a meterstick. As important, students extend their knowledge of whole number place value to decimals. In the base-ten system, each digit place represents a multiplier ten times that of the next place to the right. For example, the tens place has a value of 10 times the ones place. The hundreds place has a value of 10×10, or 100 times the ones place. Applying place value concepts to the right of the decimal, the tenths place has a value of $\frac{1}{10}$ the ones place, and the hundredths place has a value of $\frac{1}{100}$ of the ones place.

Representations and Models

Representations. In this unit, students make connections among different representations of decimals: symbols, words, physical models, and pictures. See Figure 1. Translating among these representations—drawing a shorthand picture of a collection of decimal base-ten pieces, saying numerical decimal fractions aloud, and writing a decimal number—helps students maneuver between the many contexts in which decimals appear in the real world and helps develop conceptual understanding of decimal fractions. Research indicates that students with a foundational understanding of decimals can learn computational procedures and solve problems involving decimals more effectively (Ball, 1993; Hiebert & Wearne, 1986; Hiebert, Wearne, & Taber, 1991; Lesh, Post, & Behr, 1987; National Research Council, 2001).

The Area Models. Research indicates that the area model (See Figure 1) provides a powerful context for students' early work with fractions and decimals (Behr & Post, 1992; Cramer, Post, and delMas, 2002). In particular, the use of fraction circle pieces and rectangles helps students establish ideas about equal parts, relative sizes of fractions, equivalent fractions, unit wholes, and operations on fractions. Children build on their everyday experiences (cutting toast, sharing pizza, etc.) when they use such models. Many real-life measurement situations (cooking, sewing, carpentry) involve fractions that are similar to fractions modeled with rectangles or circles. In this unit, area models are adapted specifically for decimal fractions. Fraction circle pieces are used to build number sense of decimals to the hundredths place. Square grids are used to represent decimals to the thousandths. These represenations also help students better represent the relationship between place values.

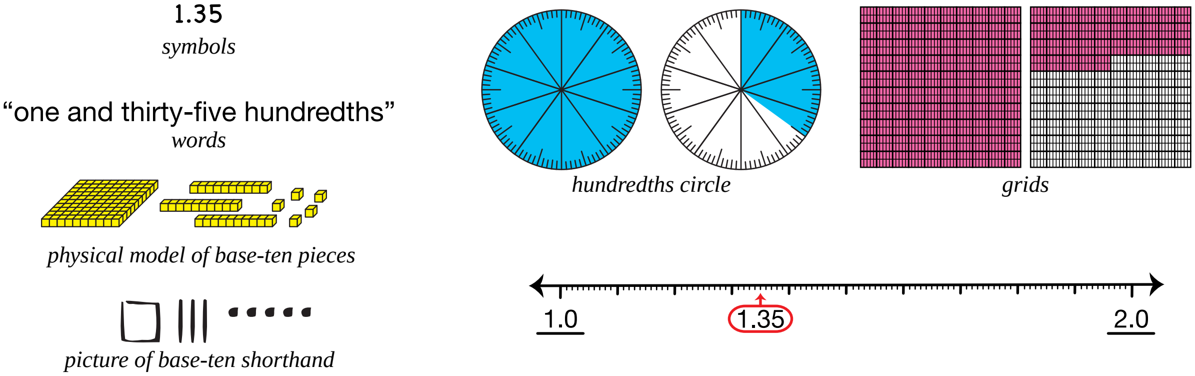
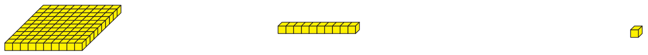
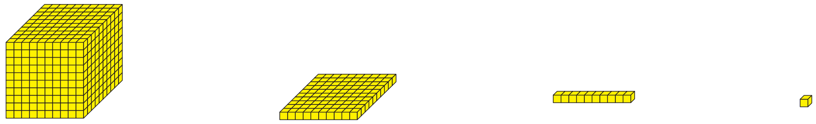


Figure 1: *The decimal 1.35 using a variety of representations*

Base-ten Pieces. Students also use base-ten pieces to model decimals but with an important change. When modeling whole numbers with base-ten pieces, the unit whole is represented as a bit. When modeling decimals, however, the unit whole needs to be defined. See Figure 2. Students have completed many activities in which the size of the whole changes, including lengths on a number line and sizes of pizza slices, so this change is a natural extension of their understanding that the size of the whole changes in varied contexts. Using base-ten pieces in this way provides a connection point to existing understandings about place value with whole numbers. (Strictly speaking, base-ten pieces are a *volume* model for decimals and not an area model, but they can be interpreted as an area model when drawn on paper.)



If a flat represents 1, then a skinny represents 0.1 and a bit 0.01.



If a pack represents 1, then a flat represents 0.1, a skinny 0.01, and a bit 0.001.

Figure 2: *Using base-ten pieces to represent decimals*

The Number Line. When a decimal fraction is represented on a number line, it is understood to occupy a unique location along a continuous scale. Students' familiarity with ruler measurement, number lines, and base-ten hoppers makes this a useful model for understanding decimals. In this unit, students represent decimals on number lines by measuring lengths in meters, centimeters, and millimeters. Students also locate decimal fractions on scaled number lines and move "decimal hoppers" between number-line locations. Students also use benchmarks to locate decimals on a number line and analyze how the location will change if the benchmarks are changed. See Figure 3. These activities establish number sense about how moving 1 unit along a number line compares with moving 0.1 units or 0.01 units.

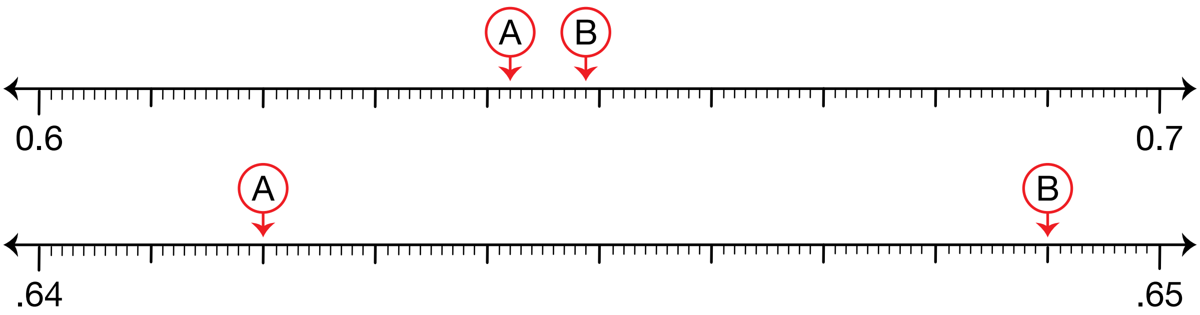


Figure 3: *0.642 and 0.649 represented on number lines using different benchmarks*

Students may have misconceptions about fractions on number lines because the focus shifts from an *amount* of area to a *position* on a number line. For example, in trying to locate $\frac{1}{2}$ inch on a ruler, a child might become confused by thinking about $\frac{1}{2}$ of the whole ruler. Furthermore, on a ruler or number line, the position is marked at the endpoint of the length. This is a departure from the area model, where labels are typically shown in the center to represent the entire quantity of area. See Figure 4. These differences may present some difficulty at first but will deepen understanding as students grow accustomed to the features of each model.

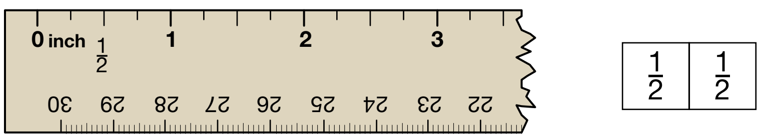


Figure 4: *One-half inch on a ruler vs. one-half of a rectangle*

Add and Subtract Decimals. Students then use these representations, everyday problem contexts, and their understanding of whole number operations to develop mental math strategies and paper-and-pencil strategies for adding and subtracting numbers with decimals. In Lesson 8, students develop a strategies menu for adding and subtracting decimals. See Figure 5 for a sample of a completed menu for subtracting decimals. Notice the menu shows both mental math and paper-and-pencil strategies.

Subtracting Decimals

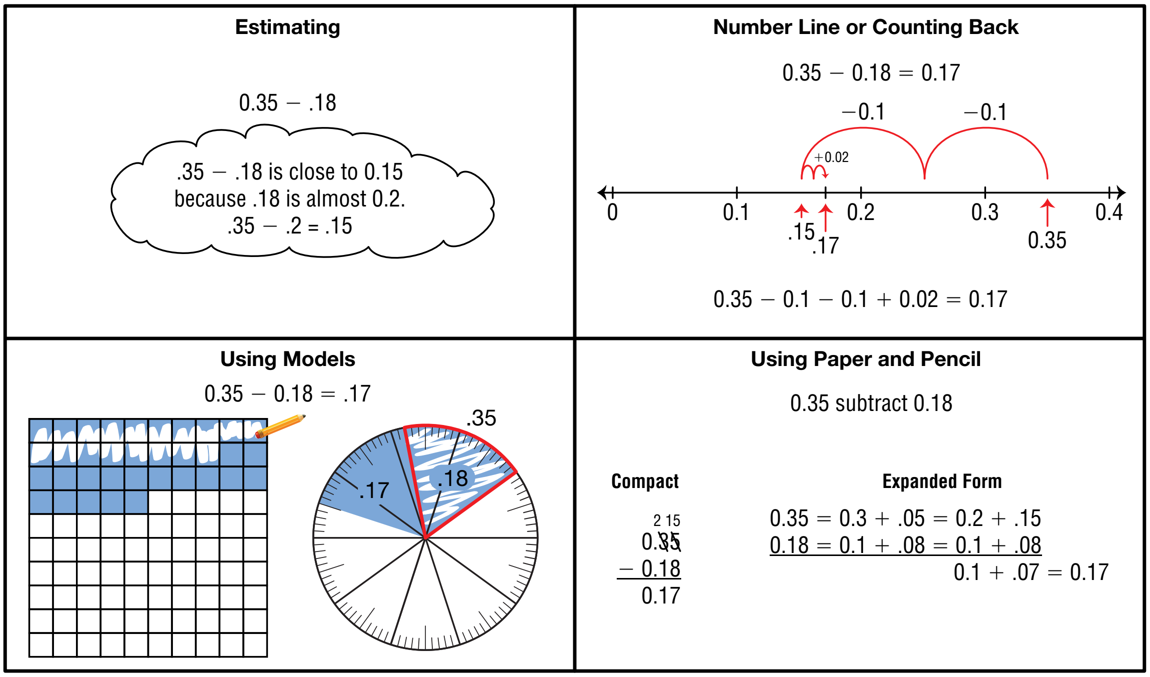


Figure 5: *Mental math and paper-and-pencil strategies for subtracting decimals*

Multiply and Divide Decimals. Students also build on their understandings of whole number computation to develop strategies for multiplying and dividing decimals. See Figure 6 for an example.

Jerome multiplied 3.4 × 4.3 this way:

	4	0.3
3	$3 \times 4 =$	$3 \times 0.3 =$
0.4	$0.4 \times 4 =$	$0.4 \times 0.3 =$

Figure 6: *Student using the rectangle model to multiply decimals*

Estimation must play an important role in developing a multiplication algorithm. With a strong understanding of place value, students will be able to use paper-and-pencil multiplication procedures they are comfortable with and then place the decimal in the correct place within the product or quotient. Even if they understand how to "count the number of places behind the decimals in the facts," in order to place the decimal, students can make mistakes if they don't estimate.

In order to estimate, students need to identify and understand the patterns between the numbers in the problem and the respective answer. For example: compare the products of 3×4 and 3×0.04. Students should be able to articulate why the product of 3×0.04 is a small number or $12 \div .04$ is a larger number.

Math Facts and Mental Math

This unit concludes the review and assessment of the multiplication and division facts to develop mental math strategies, gain proficiency, and to learn to apply multiplication strategies to larger numbers. Use the inventory in the Daily Practice and Problems to assess students' fluency.

Resources

- Ball, D.L. "Halves, Pieces, and Twoths: Constructing Representational Contexts in Teaching Fractions." In *Rational Numbers: An Integration of Research.* T.P. Carpenter, E. Fennema, and T.A. Romberg (Eds.). Lawrence Erlbaum Associates, Hillsdale, NJ, 1993.
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- Cramer, K., T. Post, and R. delMas. "Initial Fraction Learning by Fourth- and Fifth-Grade Students: A Comparison of the Effects of Using Commercial Curricula with the Effects of Using the Rational Number Project Curriculum." *Journal for Research in Mathematics Education,* 33(2), pp. 111–144, National Council of Teachers of Mathematics, Reston, VA, March 2002.
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- Wearne, D., and J. Hiebert. "Cognitive Changes during Conceptually Based Instruction on Decimal Fractions." *Journal of Educational Psychology,* 81 (4), pp. 507–513, American Psychological Association, Washington, DC, 1989.