

Nila added to the discussion: "I found out that in the mid-1800s, an astronomer named J.P. Kulik used the Sieve of Eratosthenes to find the prime numbers up to 100,000,000. He spent 20 years of his life doing this."

"I can't believe anyone would spend 20 years finding prime numbers!" exclaimed Michael.

"The sad part is that Kulik gave his manuscript to a library in Prague and they lost the sections that had the prime numbers from 12,642,000 to 22,852,800," added Mr. Moreno. "The good news is that mathematicians don't have to find large prime numbers by hand anymore. Today they use computers to do the work."

"I also found some interesting information," said John. "I found out that in 1742 a Russian mathematician named Christian Goldbach made a conjecture that every even number except 2 can be written as a sum of two prime numbers. For example, $10 = 3 + 7$."

"Right," said Mr. Moreno, "and it is called a **conjecture** because up to now no one has been able to prove or disprove it. Who knows, maybe someday one of you will be the person to do this!"

"I found a modern use for prime numbers," said Lin. "Prime numbers are used in cryptography. **Cryptography** is the study of secret codes. Some codes are based on the fact that it is hard to factor very large numbers into primes. To keep information secret, they use numbers with prime factors of 100 or more digits. One use of these codes is to protect information stored on computers. Many banks and other businesses use these codes to make sure that nobody can change or steal the information from their computers."



Use the Sieve of Eratosthenes to find all of the prime numbers between 1 and 100. Fold the 200 Chart page in the Student Activity Book in half. Follow the directions below.

1. Begin by crossing out the number 1. (Remember, 1 is not a prime number.)
2. A. The next number is 2. This is the first prime number. Circle the number 2. Cross out all of the multiples of 2 up to 100. (A **multiple** of 2 is the product of 2 and any whole number. $2 \times 1 = 2$, $2 \times 2 = 4$, $2 \times 3 = 6$, so 2, 4, and 6 are multiples of 2.)
B. Are any of the numbers you crossed out prime numbers? How do you know?

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3. A. Find the next number that is not circled or crossed out. What number is it?
B. Is this a prime number?
C. Circle this number. Cross out all of the multiples of this number up to 100.
D. Are any of these numbers prime? How do you know?
4. Continue the steps in Question 3 until you have only prime numbers left on your chart. List all the prime numbers from 1 to 100.
5. A. As you made your chart, what patterns did you see?
B. What digits do prime numbers end in?
C. Are prime numbers ever next to each other? Why or why not?
D. **Twin primes** are pairs of prime numbers that are separated by only one number. For example, 5 and 7 are twin primes. Can you find any other twin primes?
6. A. Use your sieve. Is 43 prime or composite?
B. Show or tell how you can explain your answer to someone without using your sieve. (Remember: A **prime number** is a number with exactly two factors, one and itself.)
7. A. Is 39 prime or composite?
B. Show or tell how you can justify your answer using the definition of a prime number.

✓ Check-In: Questions 8-10

8. Use the 200 Chart to help you make the following two lists:
A. List the multiples of 3 from 30 to 60.
B. List the multiples of 7 from 35 to 98.
9. List all the factors of the numbers below. You may use your lists from Question 8 and a calculator to help you.
A. 37 B. 42 C. 51 D. 53
10. Which of the numbers in Question 9 are prime? Justify your answers using the definition of a prime number.



Continue your investigation of prime numbers by finding all the prime numbers between 101 and 200.

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*Answers and/or discussion are included in the lesson.

Student Guide

Sifting for Primes (SG pp. 430–431)

Questions 1–10

1. * Cross out the number 1. See Figure 1 in the lesson.
2. A. * Circle the number 2 and cross out its multiples up to 100. See Figure 1 in the lesson.
B. * They cannot be prime because they have at least one more factor, 2, other than 1 and the number itself.
3. A. * 3
B. * Yes; the factors of 3 are only 1 and 3.
C. * Circle the number 3 and cross out all its multiples up to 100. See Figure 1 in the lesson.
D. They cannot be prime because they have at least one more factor, 3, other than 1 and the number itself.
4. * Prime numbers from 1 to 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.

×	2	3	×	5	×	7	×	×	×
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

5. A. * Answers will vary.
B. * Prime numbers except for 2 and 5 end in 1, 3, 7, and 9.
C. * Yes; 2 and 3 are next to each other, but these are the only two. No others are next to each other because there are no even-numbered primes other than 2.
D. * 11 and 13, 17 and 19, 29 and 31, 41 and 43, 59 and 61, and 71 and 73 and twin primes.
6. A. * 43 is prime.
B. * Possible response: 43 only has 1 and 43 as factors. I know it is not divisible by 2 and 5 because it is not even and it doesn't end in 5 or 0. It is not divisible by 3 or 7 because I tried it on my calculator and it didn't produce a whole number.

7. **A.** * 39 is composite.
B. * $3 \times 13 = 39$ so it has more factors than 1 and itself.
8. A. 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60

- 9. A.** 1, 37 **B.** 1, 2, 3, 6, 7, 14, 21, 42
C. 1, 3, 17, 51 **D.** 1, 53
10. 37 and 53 are prime because they have only two factors.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

- B.** 35, 42, 49, 56, 63, 70, 77, 84, 91, 98

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

Homework (SG p. 431)

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, and 199

Prime Numbers Between and 200

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

* Answers and/or discussion are included in the lesson.