Unit 10

MATHEMATICS IN THIS UNIT
Fraction Operations

Taken from the *Math Trailblazers* digital Teacher Guide

> "Instructional practices that tend toward premature abstraction and extensive symbolic manipulation lead students to have severe difficulty in representing rational numbers with standard written symbols and using the symbols appropriately."
>
> National Research Council, 2001, p. 234

Learning Progression. Students have been developing strategies for representing, comparing, and finding equivalent fractions. In this unit, students extend these representations and strategies to develop models and methods for adding, subtracting, multiplying, and dividing fractions.

Students use fraction circle pieces, rectangles models (paper folding), number lines, and drawings to represent these operations. Children need exposure to a variety of concrete models and mathematical interpretations of fractions in order to understand the underlying concepts before they can develop meaningful procedures with symbols (Cramer et al., 1997). Therefore, in this unit, students focus on developing models for the operations involving fractions.

Research suggests that although students are able to draw on their informal or "real-life" knowledge of fractions to solve problems, premature attempts at using symbolic procedures can interfere with the construction of meaningful algorithms (Mack, 1990). For example, when students are presented with a problem such as $\frac{1}{2} + \frac{1}{4}$, their answers are often incorrect because they try to add numerators and denominators based on a whole-number algorithm. The same students are often able to solve the same problem correctly when it is presented in a context, such as:

Ana ate $\frac{1}{2}$ of a pizza for dinner and another $\frac{1}{4}$ of the pizza for a snack. How much of the whole pizza did she eat?

They use their knowledge of the real world to think through the problem using context and imagery, and then find a solution that makes sense to them. That said, at this point, many students develop paper-and-pencil strategies for the operations. Encourage students to connect these procedures or patterns to the problem situation by representing the problem.

Connect Strategies. During this unit, students will develop multiple strategies for all the operations and organize these strategies into strategies Menus. See Figure 1.

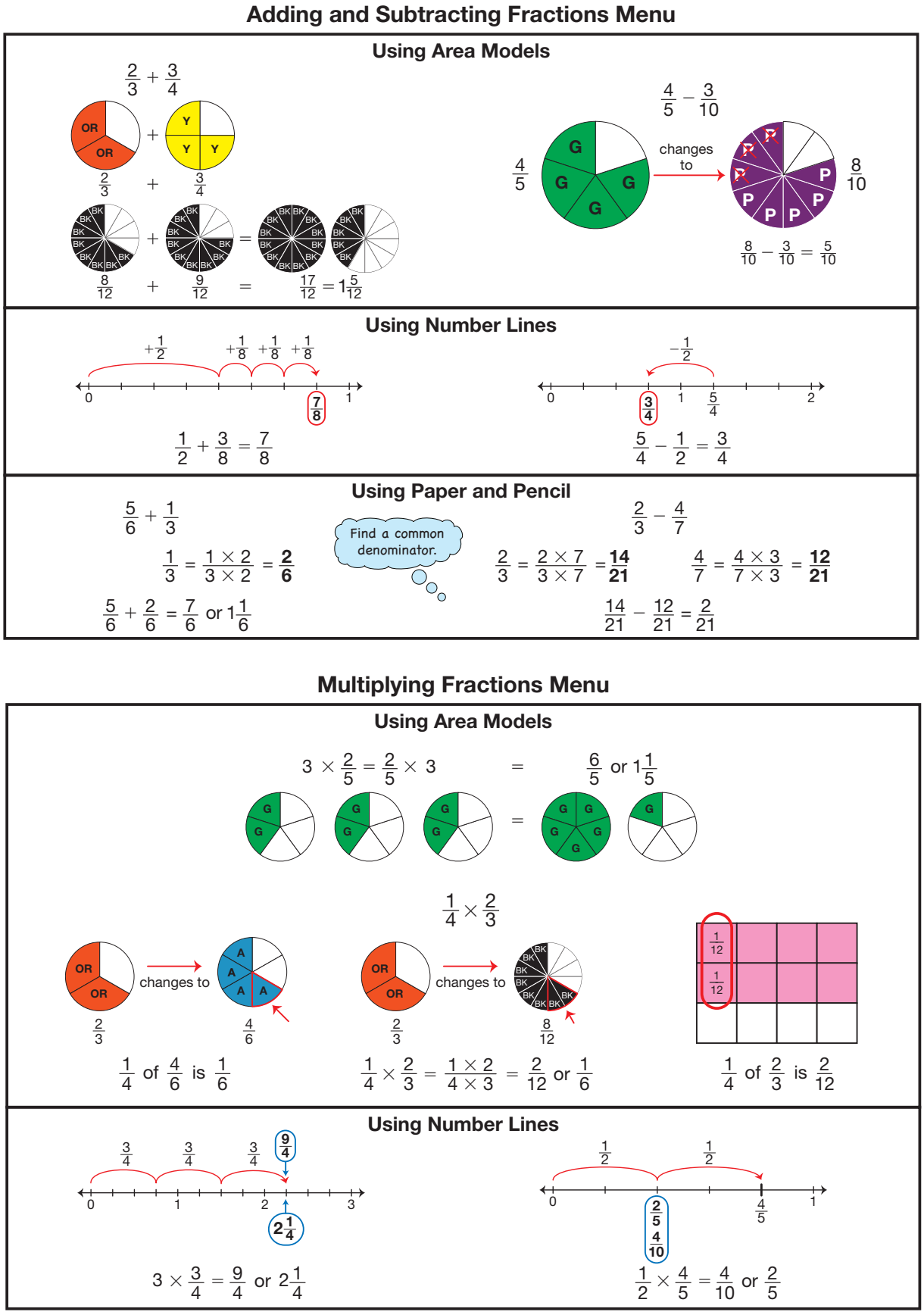

Figure 1: *Strategies Menus for Adding, Subtracting, and Multiplying Fractions*

Estimate and Check for Reasonableness. Students should be encouraged to use multiple strategies to solve a problem and estimate to check if their response is reasonable. The multiple steps involved in paper-and-pencil strategies often yield inaccurate answers. Paper-and-pencil strategies can also complicate a problem that can be solved with a simple mental math strategy. In this unit, students are encouraged to use benchmarks to estimate sums and differences.

To estimate products and quotients, students are encouraged to represent the problem with a model or context. To this end, students will also analyze patterns in products and quotients. For example: When solving 3×4 the product is larger than the factors but when solving $3 \times \frac{1}{4}$ the product is smaller than 3. One way to think of this problem is to think about having 3 groups that are the size of $\frac{1}{4}$ of a whole. These patterns and estimation strategies will help students develop flexible paper-and-pencil as well as efficient mental math strategies for solving problems involving fractions.

Resources

- Cramer, K., M. Behr, T. Post, and R. Lesh. *Rational Number Project: Fraction Lessons for the Middle Grades - Level 1* <http://cehd.umn.edu/rationalnumberproject/rnp1.html>. Kendall/Hunt Publishing Co., Dubuque, Iowa, 1997.
- Cramer, K., T. Post, and R. delMas. "Initial Fraction Learning by Fourth- and Fifth-Grade Students: A Comparison of the Effects of Using Commercial Curricula with the Effects of Using the Rational Number Project Curriculum." *Journal for Research in Mathematics Education,* 33(2), pp. 111–144, March 2002.
- Cramer, K., & Wyberg, T. Efficacy of different concrete models for teaching the part-whole construct for fractions. *Mathematical Thinking and Learning, 11*(4), 226-257, 2012.
- Curcio, F. R., and N. S. Bezuk. *Understanding Rational Numbers and Proportions.* National Council of Teachers of Mathematics, Reston, VA, 1994.
- Curcio, F. R. (series editor). *Curriculum and Evaluation Standards for School Mathematics, Addenda Series 5–8: Understanding Rational Numbers and Proportions.* National Council of Teachers of Mathematics, Reston, VA, 1994.
- Lamon, S. J. "Fractions and Rational Numbers." In *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers.* Lawrence Erlbaum Associates, Mahwah, New Jersey, 2006.
- Mack, N. K. "Learning Fractions with Understanding: Building on Informal Knowledge." In *Journal for Research in Mathematics Education.* Volume 21, Number 1, National Council of Teachers of Mathematics. Reston, VA, January 1990.
- National Research Council. " Developing Proficiency With Other Numbers." In *Adding It Up: Helping Children Learn Mathematics.* J. Kilpatrick, J. Swafford, and B. Findell, Eds. National Academy Press, Washington D.C., 2001.
- Post, T., I. Wachsmuth, R. Lesh, M. Behr. "Order and Equivalence of Rational Numbers: A Cognitive Analysis." *Journal for Research in Mathematics Education,* 16 (1), January 1985.
- Post, T. R. (editor). *Teaching Mathematics in Grades K–8, Research Based Methods.* Allyn and Bacon, Boston, 1992.
- *Principles and Standards for School Mathematics.* The National Council of Teachers of Mathematics, Reston, VA, 2000.
- Van de Walle, J.A. and L. Lovin. *Teaching Student-Centered Mathematics: Grades 3–5.* Pearson Education, Boston, MA, 2006.
- Wu, Hung-Hsi. Chapter 2: Fractions (Draft) (June 20, 2001; Revised September 3, 2002).