Unit 11 MATHEMATICS IN THIS UNIT
Equivalent Fractions Using Proportions
Taken from the *Math Trailblazers* digital Teacher Guide

Proportions. A major theme throughout the *Math Trailblazers*™ curriculum is the development of proportional reasoning. The goal of this unit is to bring together previously studied concepts of ratio and proportion with more formal ideas and strategies. Students have been developing many tools and strategies for solving problems that require proportional reasoning. For example, in Fifth Grade in the lab *Distance vs. Time* in Unit 5, students solved problems using data they collected on the walking speed of Fifth Graders. They organized their data in a table and graphed the data using a point graph. Sample data and a graph for the lab are shown in Figure 1.

Distance vs. Time

t Time in Seconds	*D* Distance in Yards	$\frac{\text{Distance}}{\text{time}}$ Ratio in $\frac{\text{yd}}{\text{sec}}$
2	3	$\frac{3}{2}$
4	6	$\frac{6}{4}$
6	9	$\frac{9}{6}$

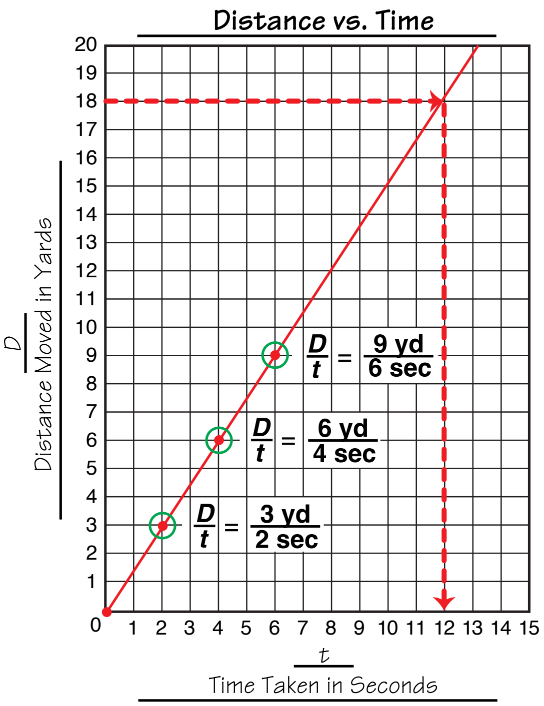

Figure 1: *In Unit 5 students used data tables and graphs to solve proportional reasoning problems about speed*

With the table and graph, students can use many strategies to solve problems such as, *"How long will it take the student to travel 18 yards?"* They can use patterns in the data table, use extrapolation on the graph, find equal ratios, or use multiplication. Students work within the context of the lab to solve more difficult problems such as, *"How far can a Fifth Grader walk in one minute? How far can a Fifth Grader walk in one hour?"*

In this unit, students use these strategies and concepts as a foundation while they develop more formal concepts and procedures for solving problems that involve proportional reasoning. They learn that a **proportion** is a statement that two ratios are equivalent.

In Lesson 2, students can review data tables and graphs from experiments and activities in previous units to formalize and extend algebraic concepts of variables in proportion. They look for similarities in the data tables and graphs and make generalizations about some of the important properties of **variables in proportion.** Two variables in an experiment are in proportion if their ratio is always the same. For example, we know that the two variables in the experiment *Distance vs. Time* are in proportion since the following properties hold:

- The ratio of distance to time ($\frac{D}{t}$) is constant for all values of distance and time. Students can see this if they write different ratios using the data points in the table. For example, $\frac{D}{t} = \frac{3 \text{ yd.}}{2 \text{ sec.}} = \frac{6 \text{ yd.}}{4 \text{ sec.}} = \frac{9 \text{ yd.}}{6 \text{ sec.}}$. (Note: Because of measurement error, experimental data is often not exact; however, the ratios will be approximately equal to one another.)
- The graph of the data is a straight line through the point (0, 0). (Note: In some experiments, due to experimental error, the data points are only close to a straight line.)
- If you multiply one of the variables by a number, the other variable increases by the corresponding factor. For example, if you double the time a student walks, the distance traveled doubles as well.

Students choose among several strategies to solve proportional reasoning problems in the unit. In a science context, the density of an object is defined as the ratio of the object's mass to the object's volume. Students apply concepts of ratio and proportion as they study the density of various materials and discover why some objects sink and others float.

Proportional reasoning is one of the important components of formal thought acquired in adolescence (Hoffer, 1992). This unit builds upon concepts and skills involving ratio and proportion that students have developed in previous grades and units and uses them as a foundation for more formal thinking. In this way, students have many avenues for solving problems that involve proportional reasoning.

Ratio of the Circumference to the Diameter. Students also discover the ratio between the diameter and the circumference of a circle. These experiences move students' thinking toward van Hiele Lesson 2. In Level 2, students explore relationships. Students informally explore the relationship between the circumference and the diameter of a circle and then use string to estimate the relationship between the lengths of diameter and circumference. Students discover that it takes approximately three diameters to "wrap around" the circumference. They find a more accurate estimate for the ratio that is called **π** or **pi**. This is the Greek letter and is pronounced "pie." Working with concrete materials enables students to gain insights into this relationship (Fuys, et al., 1988).

Approximating π. One of the earliest accurate estimates of π is credited to Archimedes circa 240 BCE. Archimedes inscribed and circumscribed polygons about a circle. Since it is easy to find the perimeter of a polygon, as opposed to finding the circumference of a circle, Archimedes was able to create a range for the value of π. As he drew polygons with more and more sides in and about the circle, he was able to get better and better estimates for π. Archimedes found that π was between $\frac{223}{71}$ and $\frac{22}{7}$. This process is described in Archimedes' *Measurement of a Circle.* Today, we often use either $\frac{22}{7}$ or 3.14 as estimates for π.

Historical Note

In 1767, Johann Heinrich Lambert showed that π is an irrational number, that is, a number that cannot be expressed as a ratio of integers. An irrational number has a nonterminating, nonrepeating decimal part. Thus, any calculation of π is only an approximation, no matter how many digits are involved. Mathematicians have found more and more of the digits in the decimal expression for π. In 1986, the NASA Ames Research Center used a supercomputer to find the value of π to 29,360,000 digits. It took the computer 28 hours to produce this estimate for π. In 1999, π was computed to over 200 billion decimal digits.

Resources

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