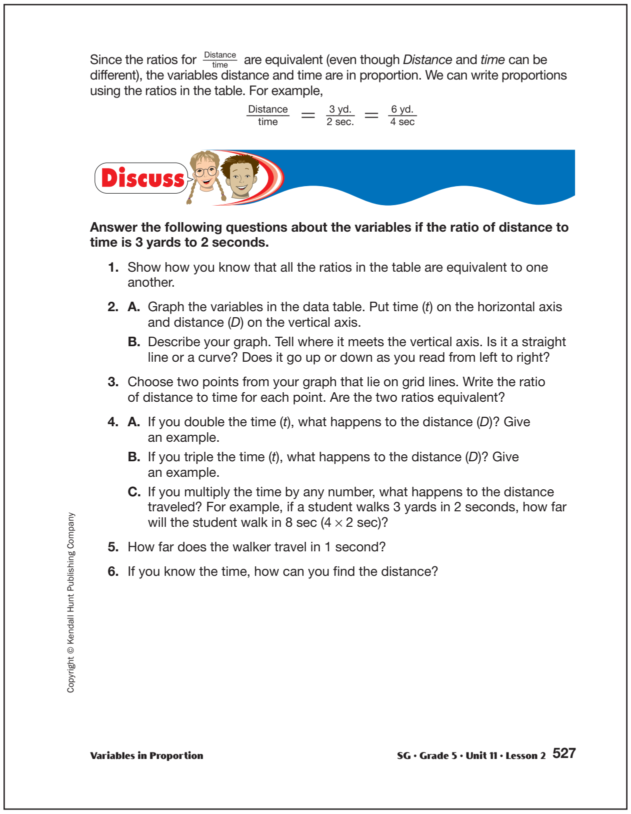

Since the ratios for $\frac{\text{Distance}}{\text{time}}$ are equivalent (even though *Distance* and *time* can be different), the variables distance and time are in proportion. We can write proportions using the ratios in the table. For example,

$$\frac{\text{Distance}}{\text{time}} = \frac{3 \text{ yd.}}{2 \text{ sec.}} = \frac{6 \text{ yd.}}{4 \text{ sec}}$$

Answer the following questions about the variables if the ratio of distance to time is 3 yards to 2 seconds.

1. Show how you know that all the ratios in the table are equivalent to one another.
2. **A.** Graph the variables in the data table. Put time (*t*) on the horizontal axis and distance (*D*) on the vertical axis.
   B. Describe your graph. Tell where it meets the vertical axis. Is it a straight line or a curve? Does it go up or down as you read from left to right?
3. Choose two points from your graph that lie on grid lines. Write the ratio of distance to time for each point. Are the two ratios equivalent?
4. **A.** If you double the time (*t*), what happens to the distance (*D*)? Give an example.
   B. If you triple the time (*t*), what happens to the distance (*D*)? Give an example.
   C. If you multiply the time by any number, what happens to the distance traveled? For example, if a student walks 3 yards in 2 seconds, how far will the student walk in 8 sec (4 × 2 sec)?
5. How far does the walker travel in 1 second?
6. If you know the time, how can you find the distance?

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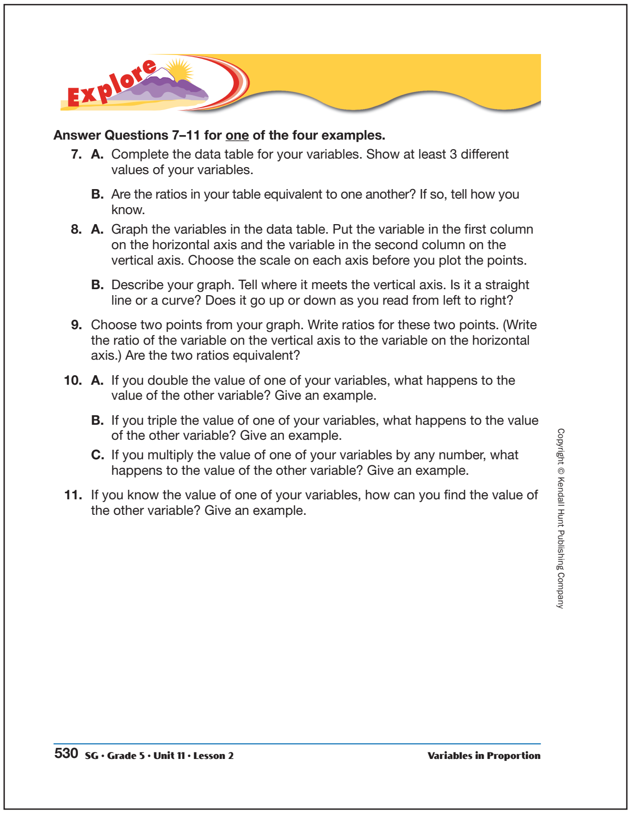

Answer Questions 7–11 for one of the four examples.

7. **A.** Complete the data table for your variables. Show at least 3 different values of your variables.
   B. Are the ratios in your table equivalent to one another? If so, tell how you know.
8. **A.** Graph the variables in the data table. Put the variable in the first column on the horizontal axis and the variable in the second column on the vertical axis. Choose the scale on each axis before you plot the points.
   B. Describe your graph. Tell where it meets the vertical axis. Is it a straight line or a curve? Does it go up or down as you read from left to right?
9. Choose two points from your graph. Write ratios for these two points. (Write the ratio of the variable on the vertical axis to the variable on the horizontal axis.) Are the two ratios equivalent?
10. **A.** If you double the value of one of your variables, what happens to the value of the other variable? Give an example.
    B. If you triple the value of one of your variables, what happens to the value of the other variable? Give an example.
    C. If you multiply the value of one of your variables by any number, what happens to the value of the other variable? Give an example.
11. If you know the value of one of your variables, how can you find the value of the other variable? Give an example.

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Student Guide

Variables in Proportion (SG pp. 527–535) Questions 1–21

1.* Since all the ratios in the table reduce to $\frac{3}{2}$, they are all equivalent.

2. **A.*** See Figure 2 in the lesson.

B.* The graph is a straight line that goes up as we read from left to right and it meets the vertical axis at (0, 0).

3.* The ratios are equivalent. Answers will vary. See Figure 2 in the lesson. Possible solutions:
4 6 y s e d c . ; 4 9 y s e c d . ; 2

4. **A.*** The distance also doubles. $\frac{3 \text{ yd.}}{2 \text{ sec.}}$; $\frac{6 \text{ yd.}}{4 \text{ sec.}}$

B.* The distance also triples. $\frac{3 \text{ yd.}}{2 \text{ sec.}}$; $\frac{9 \text{ yd.}}{6 \text{ sec.}}$

C.* If we multiply the time by any number, the distance will increase by the same factor.

$$\frac{3 \text{ yd.}}{2 \text{ sec.}} = \frac{12 \text{ yd.} \times 4}{2 \text{ sec.} \times 4} = \frac{12 \text{ yd.}}{8 \text{ sec.}}$$

5.* $\frac{1.5 \text{ yd.}}{8 \text{ sec.}}$

6.* One way to find this answer is to find the distance traveled in one second and then multiply this by the number of seconds traveled.

7. **A.*** See Figure 3 in the lesson.

B.* The ratios are equivalent since they all reduce to the same fraction.

8. **A.*** See Figure 4 in the lesson.

B. All the graphs are straight lines that go up as we read from left to right and they all meet the vertical axis at the point (0, 0).

9. Answers will vary. The ratios should be equivalent.

*Answers and/or discussion are included in the lesson.

10. A.* For all pairs of variables that are in proportion, if we double the value of one of the variables, the value of the other variable will also double. In Example 1, if we double the distance, the time will also double.

B.* For all pairs of variables that are in proportion, if we triple the value of one of the variables, the value of the other variable will also triple. In Example 1, if we triple the distance, the time will also triple.

C.* For all pairs of variables that are in proportion, if we multiply one of the variables by any number, the value of the other variable increases by the same factor. In Example 4, since the ratio of peanuts to sugar is $\frac{1}{3}$, if we use 10 cups of peanuts, we must use $10 \times 3 = 30$ cups of sugar.

11.* If we know the value of one of the variables, we can use patterns in the table to find the value of the other variable. In Example 1, we can find the distance by multiplying the time by 2. We can also set up a proportion such as $\frac{D}{T} = \frac{6}{3} = \frac{?}{9}$.

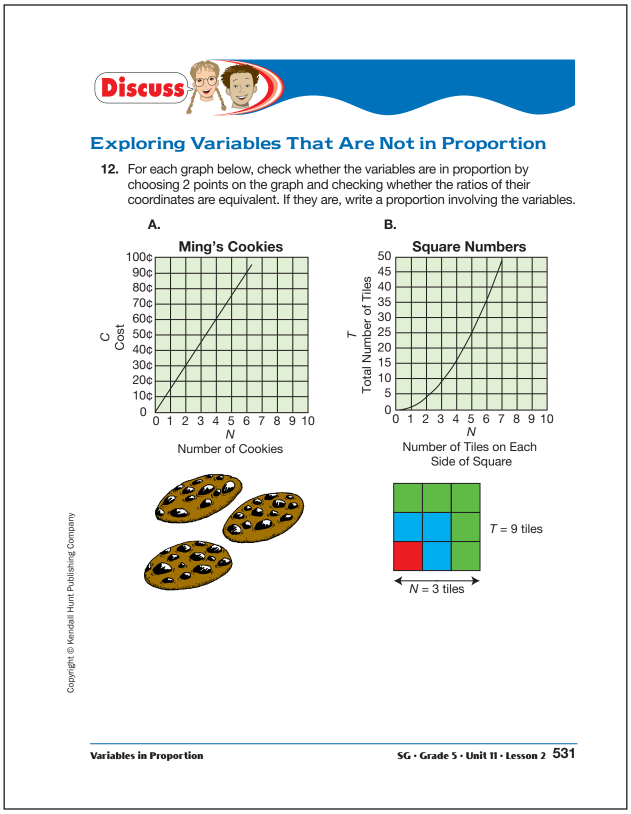
12. A.* Choosing the points (2 cookies, 30¢) and (4 cookies, 60¢), the ratios $\frac{30}{2}$ and $\frac{60}{4}$ are equivalent fractions. Therefore, the variables *N* and *C* are in proportion.

B.* Choosing the points (4, 16) and (5, 25), the ratios $\frac{16}{4}$ and $\frac{25}{5}$ are not equivalent fractions. Therefore, the variables *N* and *T* are not in proportion.

C.* Choosing the points (2, 8) and (4, 12), the ratios $\frac{8}{2}$ and $\frac{12}{4}$ are not equivalent fractions. Therefore, the variables *N* and *C* are not in proportion.

D.* Choosing the points (2, 6) and (3, 9), the ratios $\frac{6}{2}$ and $\frac{9}{3}$ are equivalent fractions. Therefore, the variables *Y* and *F* are in proportion.

E.* Choosing the points (8, 12) and (9, 4), the ratios $\frac{12}{8}$ and $\frac{4}{9}$ are not equivalent fractions. Therefore, the variables *N* and *M* are not in proportion.



Discuss

Exploring Variables That Are Not in Proportion

12. For each graph below, check whether the variables are in proportion by choosing 2 points on the graph and checking whether the ratios of their coordinates are equivalent. If they are, write a proportion involving the variables.

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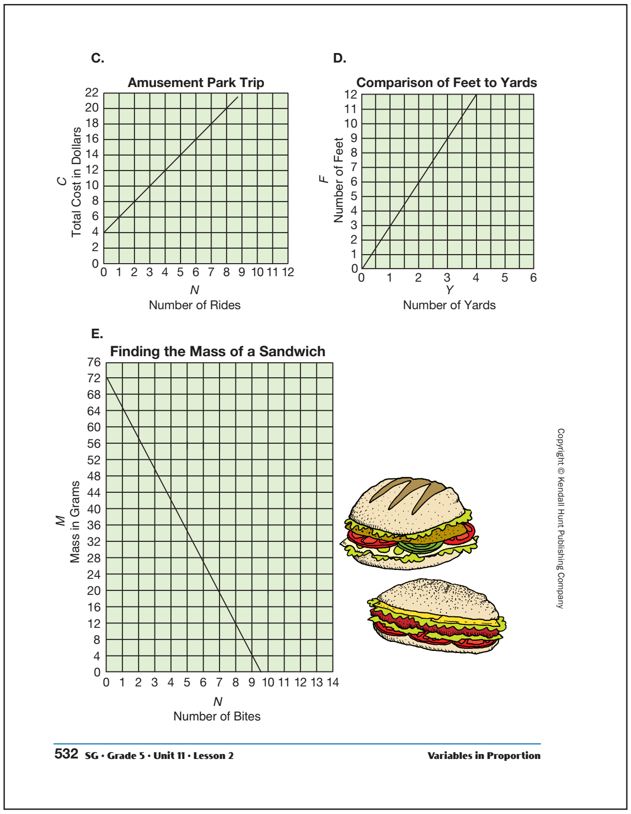

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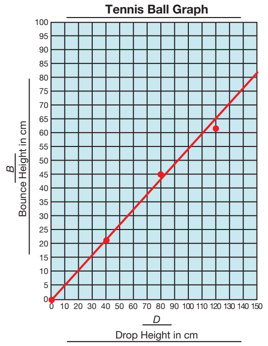
*Answers and/or discussion are included in the lesson.

13. Describe two features that graphs of variables in proportion have in common.

14. Choose one of the graphs from Question 12 that represents variables that are not in proportion. Choose a value of one of the variables and double it. Does the corresponding value of the other variable double? Show your example. How is this different from variables that are in proportion?

Using Graphs to Solve Variables in Proportion Problems

John dropped a tennis ball from three different drop heights (*D*), 40 cm, 80 cm, and 120 cm, and measured how high it bounced (*B*). He made the following graph of his data.



John and Felicia wondered how high the ball would bounce if it were dropped from *D* = 200 cm, but the numbers they wanted are not on the graph. They thought of two different ways to solve this problem:

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Method 1. John used his graph to set up a proportion involving the variables *B* and *D,* and then he solved the proportion.

John saw that point *D* = 100 cm, *B* = 54 cm is on his line. Using this information, he wrote the proportion:

$$\frac{B}{D} = \frac{54\text{ cm}}{100\text{ cm}}$$

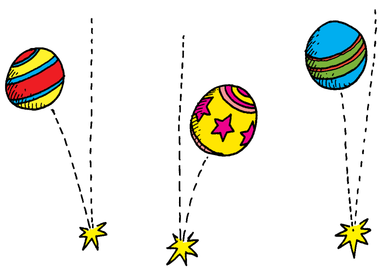
Since the graph is a straight line through (0, 0), John knew that the ratio $\frac{B}{D}$ would be equivalent for all points on the line, including the point he wondered about: *D* = 200 cm. So, John wrote the proportion:

$$\frac{B}{200\text{ cm}} = \frac{54\text{ cm}}{100\text{ cm}}$$

15. **A.** Solve John's proportion to find how high the ball would bounce if it were dropped from 200 cm.
    B. Write and solve a proportion to find the height John would have to drop the ball for it to bounce 162 cm.

Method 2. Felicia thought of another method to find how high a ball would bounce if it were dropped from 200 cm. She read on the graph that *B* = 54 cm when *D* = 100 cm. Since the variables are in proportion, she knew that doubling *D* would cause *B* to double too. Therefore, the bounce height would be *B* = 2 × 54 cm = 108 cm.

16. Use Felicia's Method to find how high the ball would bounce if it were dropped 400 cm.



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13.* The graphs of variables in proportion are straight lines and they always meet the vertical axis at the point (0, 0).

14.* Using Graph C, for 2 rides the total cost is 8 dollars. If we double the number of rides to 4 rides, we can see from the graph that the total cost is only 12 dollars. Doubling the number of rides did not double the cost. Therefore, the variables *N* and *C* are not in proportion.

15. **A.*** $B = 108$ cm

    B.* $\frac{B}{D} = \frac{162\text{ cm}}{?} = \frac{54\text{ cm}}{100\text{ cm}}$; ? = 300 cm

16.* $B = 4 \times 54$ cm = 216 cm

17. About 45 cm

18. 100 cm; $\frac{54\text{ cm}}{100\text{ cm}}$ or 75:100

19.* Possible responses: Since the graph is a straight line going through (0, 0) the variables *D* and *B* are in proportion. Since the ratios $\frac{45}{60}$ and $\frac{75}{100}$ both reduce to the same fraction, $\frac{3}{4}$, the variables *D* and *B* are in proportion.

20. **A.** $\frac{B}{D} = \frac{?}{240\text{ cm}} = \frac{45\text{ cm}}{60\text{ cm}}$; ? = 180 cm

    B. Since 240 = 60 × 4, *B* = 45 × 4 = 180 cm.

21.* $\frac{B}{D} = \frac{75\text{ cm}}{100\text{ cm}} = \frac{?}{150\text{ cm}}$; ? = $112\frac{1}{2}$ cm or since *B* is about 37 cm when *D* is 50 cm, the bounce height will be about 3 × 37 or about 111 cm for a drop height of 150 cm.

John's Super Ball Experiment

✓ Check-In: Questions 17-21

John repeated his experiment using a super ball. Below is a graph of his data.

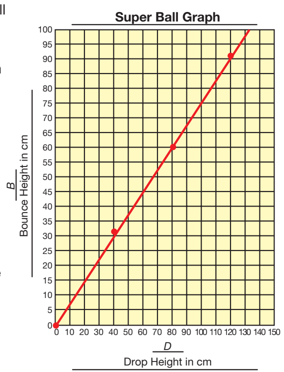
17. Predict how high the super ball would bounce if it is dropped from a height of 60 cm.

18. From what height should John drop the ball if he wants the ball to bounce 75 cm? Write your solution as a ratio.

19. Are the variables drop height (*D*) and bounce height (*B*) in proportion? How do you know?

20. Predict how high the ball will bounce if it is dropped from a height of *D* = 240 cm.
    A. Use John's Method to solve the problem and write a proportion. (*Hint:* $\frac{B}{D} = \frac{?}{?}$)
    B. Use Felicia's Method to solve the problem.

21. Predict how high the ball will bounce if it is dropped from *D* = 150 cm. Explain your reasoning.



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*Answers and/or discussion are included in the lesson.

Homework (SG p. 536)
Questions 1–6

1. A. $150.00; $\frac{150}{100}$
   B. 150 laps; $\frac{225}{150}$
   C. $300.00; $\frac{300}{200}$
   D. 250 laps; $\frac{375}{250}$
2. between 330–350 laps
3. About $240–$275
4. $\frac{\$7.50 \text{ dollars}}{5 \text{ laps}}$
5. 1000 laps
6. 10 laps

Homework

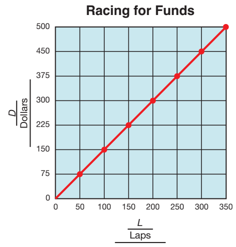
Solve the problems. Refer to the table or graph to answer the questions.

The students in Mr. Moreno's class are going to run laps around the school to raise money for their class trip to the nature preserve. For every 10 laps the students complete, the neighborhood sports store will donate $15.00 to the class fund. The students made a graph and table to see how much money they could raise by running laps.

1. Copy and complete the table.

Racing for Funds

	L Number of Laps	*D* Dollars Raised	$\frac{D}{L}$ Ratio
	10	15	$\frac{15}{10}$
	50	75	$\frac{75}{50}$
A.	100		
B.		225	
C.	200		
D.		375	
	300	450	$\frac{450}{300}$



2. Students want to explore the Bat and Cave exhibit at the nature preserve. It costs $500. About how many laps would students have to finish to cover the cost?
3. The first group of students ran 170 laps. About how much money have they raised so far?
4. What is the ratio for laps to dollars if one student finishes 5 laps?
5. The students raised enough money for the field trip, but now they want to raise $1500 to contribute to the Save the Gray Bat Fund. How many laps will they need to complete? Use the ratios to help you solve this problem. Show your work.
6. There are 100 students in 5th grade. How many laps will each student need to complete for the 5th grade to raise $1500? Show your work.

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